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Short pulse characterization requires recognizing inseparability of autocorrelation and spectral measurements

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Abstract

We present a mathematical approach to appreciate that short pulse characterization requires recognizing inseparability of the measurements of the amplitude envelope-correlation and spectral measurements and then use a suitable iterative approach to derive the real characteristics. We will use a standard Michelson interferometer, as usual, to introduce the autocorrelation function of a pulse containing single and multiple frequencies. In the process, we also underscore that detectors play the key role in generating measurable Superposition Effects (SE), recognized as fringes after the detectors carry out the square modulus operation. Simple mathematical summation of amplitude factors, the Superposition Principle (SP), is not directly observable. We underscore this by mentioning that we present EM waves as classical and detectors as quantum mechanical. This semi-classical approach has been established by Lamb and Jaynes, which is indirectly supported by Glauber's comment, "A photon is what a detector detects". The semi-classical approach helps us separate the phenomenological difference between the absorbed detected energy by a detector (SE) from the energy supplied by the simultaneously present multiple wave amplitudes (SP). As in atomic and molecular physics, we use the detector's dipolar stimulation as the product of its linear dipolar polarizability multiplied by all the EM fields stimulating it simultaneously. The analysis also demonstrates that for a pulse containing multiple frequencies, the two-beam autocorrelation function becomes a product of the traditional amplitude correlation factor and a frequency-comb correlation factor. Hence, the spectral interpretation of a short pulse and two-beam autocorrelation are inseparable. Therefore, the detailed characterization would require iterative computational approach by guessing the most plausible functional forms. This deeper understanding can be applied to rapid re-calibration of pulsed lasers that need to be maintained at single mode but has the tendency to move to multimode behavior. If the newly measured autocorrelation function differs from the original amplitude correlation factor, then one should check for the spectral characteristics first, before assuming that only the pulse shape has changed.

Keywords: Pulse characterization; Amplitude envelope-correlation factor; Frequency comb-correlation factor; Measurable Superposition Effect; Mathematical Superposition Principle; Non-Interaction of Waves (NIW); Coherence; Spectrometry.

1. Introduction

We present a mathematical analysis to demonstrate that the two-beam autocorrelation function for a laser pulse containing multiple longitudinal modes (frequency-comb) is a product of the traditional amplitude correlation factor and a frequency-comb correlation factor. Analytically we use the semi-classical approach established by Lamb [1] and Jaynes [2,3] and indirectly supported by Glauber comment, "A photon is what a detector detects" [4]. This approach separates the conjoint stimulation provided by the n classical waves $E_n(\nu_n)$ (Eq.1) to the detector; whose quantum properties are contained in its linear dipolar polarizability $\chi(\nu)$, with ν_n being the resonant band of quantum transition frequencies.

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$$\Psi = \sum_n \chi_n(\nu) E_n(\nu_n) \tag{1}$$

Then the frequency resonant quantum detectors carry out the non-linear quadratic process step, $d \equiv \Psi^* \Psi$, where d is the released photo electric current by the detector [5,6].

$$d \equiv \Psi^* \Psi = \left| \sum_n \chi_n(\nu) E_n(\nu_n) \right|^2 \tag{2}$$

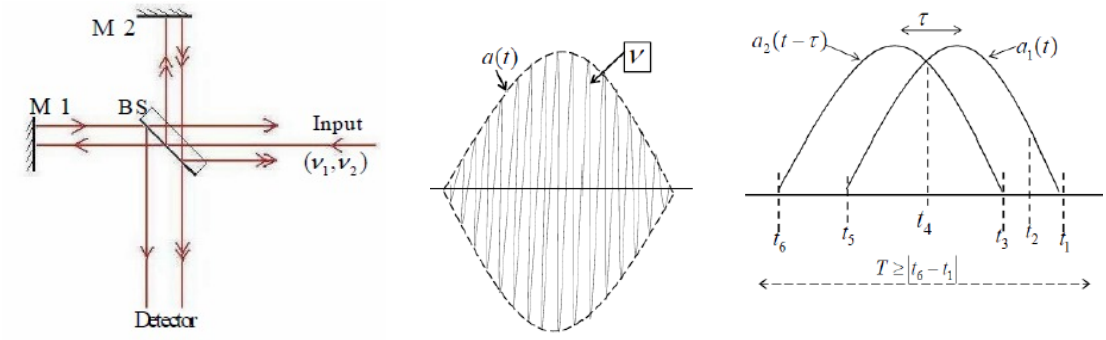
Please, note that the energy re-distribution, or the fringes due to Superposition Effect (SE), implicated by this quadratic Eq.2, becomes manifest only after the detector array absorbs energy out of all the simultaneously stimulating EM fields impinged on the detector. The quantumness in the light detection data emerges through the quantum properties of the detecting dipoles, which are embedded in its quantum characteristics $\chi(\nu)$. If we have a very narrow band of frequencies, then $\chi(\nu)$ can be replaced the detector *constant* χ^2 and taken out of the two operations represented by the summation and the square modulus, as per mathematical rules:

$$d \equiv \Psi^* \Psi = \chi^2 \left| \sum_n E_n(\nu) \right|^2 \tag{3}$$

For this narrow-band case, Eq.3 will still validate the measured data (scientific evidence), however, the equation may imply that the field amplitudes by themselves $\left| \sum_n E_n(\nu) \right|^2$ can carry out the consecutive operations of (i) first re-organizing the amplitude distribution through self-summation and (ii) then re-organize the quadratic energy distribution accordingly (intensity fringes). Electromagnetic wave amplitudes, by themselves, do not possess such capability. The detectors carry out these operations. That is why we can never register *Superposition Effect* without inserting a detector array within the volume of superposed light beams. Energies of EM waves can be altered only through classical and/or quantum mechanical interactions with materials [7-9].

2.1 Two-beam autocorrelation function for a pulse with a single carrier frequency

The detailed history and mathematical formalism behind the evolution of the concept of coherence, or autocorrelation function between a pair of replicated fields, can be found in references [10,11]. To formulate the 2-beam autocorrelation, we use some form of a two-beam interferometer, say, a Michelson Interferometer (MI) (see Fig.1a) and a single input pulse with a single carrier frequency (see Fig.2b). For mathematical simplicity, we assume that the pulse is clipped off from a CW single frequency laser by a suitable electro-optic gating device.



(a) (b) (c)

Figure 1: (a) A Michelson interferometer (MI). (b) The incident pulse envelope and its carrier frequency. (c) The MI generates two replicated pulses with a relative delay out of a single incident pulse. These pair of pulses are partially superposed and jointly stimulates a detector with time varying amplitudes.

The MI replicates an initial single pulse into a pair of pulses with a relative temporal delay τ as $a(t)\exp[-i2\pi\nu t]$ and $a(t-\tau)\exp[-i2\pi\nu(t-\tau)]$ (see Fig.1c). Then the time varying detector current will be proportional to:

$$\begin{aligned} d(t,\tau) &= \left| a(t)\,\mathrm{e}^{-i2\pi\nu t} + a(t-\tau)\,\mathrm{e}^{-i2\pi\nu(t-\tau)} \right|^2 \\ &= [a^2(t) + a^2(t-\tau)] + 2a(t)a(t-\tau)\cos 2\pi\nu\tau \\ &\equiv C_1(t)[1 + C_2(t)\cos 2\pi\nu\tau] \end{aligned} \tag{4}$$

$$\begin{aligned} &\text{Where, } C_1(t) \equiv [a^2(t) + a^2(t-\tau)] \equiv 2a^2(t); \\ &\text{and } C_2(t) \equiv 2a(t)a(t-\tau)\big/[a^2(t) + a^2(t-\tau)] \equiv a(t)a(t-\tau)\big/a^2(t) \end{aligned} \tag{5}$$

Here $a^2(t)$ represents the instantaneous intensity. The time dependent excursions of the two-beam cosine fringes, $C_2(t)\cos 2\pi\nu\tau$ (Eq.2), is due to the time varying excitations of the detector caused by the displaced pair of pulses $a(t)$ & $a(t-\tau)$ (Fig.1c). During the intervals, t_1 to t_3, and t_5 to t_6, there are no real physical superposition effect on the detector due to the absence of overlap between the pulses. The detector array will register temporally varying intensity due to one pulse only without any fringes during these initial and final periods. Even during the interval t_3 to t_5, the fringe visibility will vary with time as the detector experiences time varying joint amplitude stimulations. Only at the instant t_4, the fringe visibility will *momentarily* have the perfect unit visibility because the two amplitudes are exactly equal. A ps-streak camera, with an incident ns pulse, should be able to record this time evolving fringe visibility. However, if we use a long time integrating photographic plate; or a digital camera with an exposure, time $T \geq (t_6 - t_1)$, we can register time integrated fringe visibility, ${}^{nrm.}D(\tau)$ as given below:

$${}^{nrm.}D(\tau) \equiv \frac{1}{\mathrm{E}_{\mathrm{pls.}}} \int_{T>(t_6-t_1)} d(t,\tau)\, dt = [1 + \gamma_a(\tau)\cos 2\pi\nu\tau] \tag{6}$$

$$\text{Where,} \quad \gamma_a(\tau) \equiv \left[\int a(t)a(t-\tau)\, dt \Big/ \int a^2(t)\, dt \right] \tag{7}$$

Here, $\gamma_a(\tau)$ represents the time-integrated normalized autocorrelation function and $\mathrm{E}_{\mathrm{pls.}}$ is the total energy contained in the incident pulse. The subscript a on $\gamma_a(\tau)$ underscores that its variation derives from the time varying amplitude envelope $a(t)$, and not due to the physical existence of mathematical Fourier frequencies $\tilde{a}(f)$, the Fourier transform of the envelope function $a(t)$ (see Eq.10 below). Mandel and Wolf [10,11] rigorously show that the modulus of the autocorrelation function is the traditionally measured visibility of fringes as defined by Michelson:

$$V(\tau) = ({}^{nrm.}_{\max.}D - {}^{nrm}_{\min.}D)\big/({}^{nrm.}_{\max.}D + {}^{nrm.}_{min.}D) \equiv \left|\gamma_a(\tau)\right| \tag{8}$$

From the auto correlation theorem, we also know that the normalized autocorrelation function and the Fourier spectral density function form a Fourier transform pair [10,11]:

$$\gamma_a(\tau) = \int {}^{nrm.}\tilde{A}(f)\, e^{i2\pi f\tau} df; \qquad {}^{nrm}\tilde{A}(f) = \int \gamma_a(\tau)\, e^{-i2\pi f\tau} d\tau \tag{9}$$

Note that ${}^{nrm.}\tilde{A}(f)$ is the normalized $|\tilde{a}(f)|^2$ (see Eq.7). We have used f to denote Fourier frequencies instead of ν, to differentiate between mathematical Fourier frequency f from the physical emission frequency ν of EM wave packets from atoms and molecules. This is meant to underscore that, while the Fourier theorem is mathematically correct and is a powerful tool for diverse analysis, linear optical systems cannot execute complex Fourier algorithm of finite amplitude entering into a *linear optical system* with a finite velocity c.Let us recall that we use *nonlinear materials* to alter primary optical frequencies. Mathematically, the amplitude envelope $a(t)$ and its Fourier amplitude spectrum are related by the following standard Fourier transform pair:

$$a(t) = \int \tilde{a}(f)\, e^{+i2\pi ft} df; \qquad \tilde{a}(f) = \int a(t)\, e^{-i2\pi ft} dt \tag{10}$$

Note also that the Fourier conjugate variables in Eq.9 are $\{f \ \& \ \tau\}$, but those for Eq.10 are $\{f \ \& \ t\}$. The parameters $t \ \& \ \tau$ and $f \ \& \ \nu$ are not interchangeable as they have *different physical origins and meanings*. This is not underscored in the prevailing texts. The parameter τ is the measurable physical delay we introduce in our optical devices, which is different from the running clock time t. Similarly, f is the mathematical Fourier's frequency and ν is the physical source frequency, which are usually determined by quantum transition in atoms and molecules. Appreciation of these distinctions between different mathematical parameters helps us make causal interpretations of the phenomenon under consideration when experiments validate a proposed theory [8].

2.2 Two-beam autocorrelation for a pulse with two and more carrier frequencies

In the prevailing field of pulse laser technology, the presence of multiple frequencies (frequency comb) is quite common. In this section, we derive the autocorrelation function for such a pulse following Michelson's approach [12] to Fourier transform spectroscopy for broad and multi-line spontaneous emissions from different elements. We will simplify our mathematics assuming we are clipping off a single pulse, using an electro-optic switch, from a CW laser running in two longitudinal modes of *equal amplitude*. This normalized two-frequency spectral density function can be thought of as the sum of a pair of delta functions:

$$_{\nu,2}s(\nu) = (1/2)[\delta(\nu - \nu_1) + \delta(\nu - \nu_2)] \tag{11}$$

Then the instantaneous detector current, due to replication through the Michelson interferometer, can be represented by:

$$d(t,\tau) = \left| a(t)\{e^{-i2\pi\nu_1 t} + e^{-i2\pi\nu_2 t}\} + a(t-\tau)\{e^{-i2\pi\nu_1 (t-\tau)} + e^{-i2\pi\nu_2 (t-\tau)}\} \right|^2 \tag{12}$$

As per Michelson, different frequencies are "incoherent" and hence they do not "interfere". In reality, with modern high-speed detector, one can register the beat signal, represented by the cross term in Eq.12. Therefore, let us assume that we are using a very slow detector that averages out the high frequency oscillatory heterodyne current into a mean DC current. Then, we can re-write Eq.9 by re-grouping the terms separately for ν_1 and ν_2, as:

$$d(t,\tau) = \left| a(t) e^{-i2\pi\nu_1 t} + a(t-\tau) e^{-i2\pi\nu_1 (t-\tau)} \right|^2 + \left| a(t) e^{-i2\pi\nu_2 t} + a(t-\tau) e^{-i2\pi\nu_2 (t-\tau)} \right|^2 \tag{13}$$

This is a very similar expression as in Eq.4, but a sum of two sets of intensity terms for two independent frequencies. Then, the time integrated slow detector current due to replicated two-frequency pulse would be very similar to Eq.6, but a sum of two separate frequency terms. Note that to maintain normalization of total energy, each mode (frequency) now should be assigned half the pulse energy, assuming equal:

$$\begin{aligned} _{\nu,2}D(\tau) &= (1/2)E_{\text{pls.}}[1 + \gamma_a(\tau)\cos 2\pi\nu_1\tau] + (1/2)E_{\text{pls.}}[1 + \gamma_a(\tau)\cos 2\pi\nu_2\tau] \\ &= (1/2)E_{\text{pls.}}[2 + \gamma_a(\tau)\{\cos 2\pi\nu_1\tau + \cos 2\pi\nu_2\tau\}] \\ &\equiv (1/2)E_{\text{pls.}}[2 + \gamma_a(\tau)\gamma_{\nu,2}(\tau)] \end{aligned} \tag{14}$$

Now, normalize Eq.11 by dividing both sides by $E_{pls.}$:

$$^{nrm.}_{\nu,2}D(\tau) = \left[1 + (1/2)\gamma_a(\tau) \times \gamma_{\nu,2}(\tau)\right] \tag{15}$$

Michaelson was using steady-state spontaneous emissions; so he ignored the correlation term $\gamma_a(\tau)$ for the spontaneous emission pulses in Eq.12. His brilliance was in recognizing that by normalizing the interferogram intensity and then removing the DC term in Eq.11, he is left with the summation of pure *oscillatory* fringe function $\{\cos 2\pi\nu_1\tau + \cos 2\pi\nu_2\tau\}$. The mathematical Fourier transform gives back the original spectral density function shown in Eq.11:

$$_{\nu,2}s(\nu) = FT\left[\!\left[\gamma_{\nu,2}(\tau)\right]\!\right] \equiv FT\left[\!\left[(1/2)(\cos 2\pi\nu_1\tau + \cos 2\pi\nu_2\tau)\right]\!\right] \tag{16}$$

We see that the autocorrelation function for multi-frequency signal and the spectral density function also form a Fourier transform pair. This is the conceptual origin of Fourier transform spectroscopy. Consult [12,13] for rigorous derivations where the input signal is more complex.

We can now easily generalize normalized fringe function of Eq.15 for a pulse containing a comb of N-frequencies, provided they have been set to oscillate with *equal strength* through some intra-cavity gain equalizer:

$$^{nrm.}_{\nu}D(\tau) = \left[1 + \frac{1}{N}\gamma_a(\tau) \times \gamma_{N,\nu}(\tau)\right] \tag{17}$$

$$\text{Where, } \gamma_{N,\nu}(\tau) = \sum\nolimits_n \cos 2\pi\nu_n\tau \tag{18}$$

For a realistic frequency comb of un-equal strengths, say a Lorentzian gain envelope for a lasing medium, each one of the n-th frequency term $\cos 2\pi\nu_n\tau$ has to be multiplied by the appropriate mode intensity (weighting) factor. For a continuously broadened frequency distribution, as in Doppler broadened spontaneous emission, the cosine fringe terms must be multiplied by the Gaussian weighting function and the summation of Eq.18 should be replaced by an appropriate integral, Eq.19. So, the frequency correlation function remains the cosine Fourier transform of the spectral density function.

$$\gamma_\nu(\tau) = \int d\nu\ s(\nu)\cos 2\pi\nu\tau \tag{19}$$

The amplitude-frequency joint correlation function remains as the product of the two separate correlation functions:

$$\gamma_{a,\nu}(\tau) = \gamma_a(\tau)\int d\nu\ s(\nu)\cos 2\pi\nu\tau \tag{20}$$

Eq.19 through Eq.20 are important to extract deeper insights about the joint *amplitude-frequency* correlation function $\gamma_{a,\nu}(\tau)$ for all realistic pulsed emissions. We can now generalize $\gamma_{a,\nu}(\tau)$ as a product of the two separate correlation functions due, *first*, to the finite *amplitude* envelope of a pulse; *second*, to the distributed carrier *frequencies* contained in the pulse. We have dropped the subscript N from $\gamma_{N,\nu}(\tau)$ to accommodate the fact that it could also be due to a continuous frequency distribution function:

$$\gamma_{a,\nu}(\tau) \equiv \gamma_a(\tau)\gamma_\nu(\tau) \tag{21}$$

Using the *convolution theorem*, the Fourier transform of Eq.21 can be expanded, using all the logics behind Eq.16, as:

$$FT\left[\!\left[\gamma_{a,\nu}(\tau)\right]\!\right] \equiv FT\left[\!\left[\gamma_a(\tau)\gamma_\nu(\tau)\right]\!\right] = \tilde{\gamma}_a(f) \otimes \tilde{\gamma}_\nu(\nu) = {}^{nrm.}A(f) \otimes s(\nu) \equiv {}_{eff}S(f) \tag{22}$$

This is the generalized autocorrelation theorem for pulse containing many carrier frequencies. The effective autocorrelation function, $\gamma_{a,\nu}(\tau)$, and the effective spectral density function, ${}_{eff}S(f)$, follow the same basic logics

as for the Eq.9 and 10. Note, however, that the measured broadening of ${}_{eff}S(f)$ has happened due to the convolution between real physical frequency distribution $s(\nu)$ from the source and the mathematical Fourier frequency ${}^{nrm.}A(f)$.

Eq.15 and 17 represent the key issue of this paper that the two-beam autocorrelation function for a multi-frequency laser pulse is a product of two unknown pulse parameters, the pulse envelope-function and the frequency distributions (or the comb) function. Therefore, the detailed characterization would require iterative computational approach by guessing the most plausible functional forms for each parameter. We should note that a direct spectral analysis of the pulse by a spectrometer does not directly give us $s(\nu)$. A spectrometer gives us ${}_{eff}S(f) = {}^{nrm.}A(f) \otimes s(\nu)$, as in Eq.22 [8]. Further details will be presented in a future paper.

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