



Visualizing superposition process and appreciating the principle of non-interaction of waves

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ABSTRACT

We demonstrate the dynamic evolution of superposition effects to underscore the importance of visualizing interaction processes. The model recognizes the principle of Non-Interaction of Waves. Recordable linear fringes, bisecting the Poynting vectors of the two crossing beams, have time evolving amplitude patterns in the bright fringes because the two superposed E-vectors oscillate through zero values while staying locked in phase. If a detector registers steady, stable bright fringes, it must do so by time integration. The QM recipe to model energy exchange by taking the square modulus of the sum of the complex amplitudes has this time integration built into it. Thus, we will also underscore the importance of assigning proper physical processes to the mathematical relationships: the algebraic symbols should represent physical parameters of the interactants and the mathematical operators connecting the symbols should represent allowed physical interaction processes and the guiding force.

Keywords: non-interaction of waves, non-interference of light, superposition effect, mathematical mapping of physical processes, visualizing interaction processes

1. INTRODUCTION

We necessarily base all our knowledge, especially as summarized by our mathematical theories, on measurements of the world around us taken with various instruments. Examples of such instruments include our senses (vision, hearing, touch etc.) as well as mechanical and electronic devices we devise to extend our senses beyond the limited domain provided by our bodies.

Too often one forgets, within science as well as without, that *all* sensors first of all provide a limited range as well as limited resolution (not to mention issues with noise, accuracy and other technical issues and details). Secondly, the data resulting from such measurements represent, at best, a lossy *transformation* of what one set out to measure. The authors find this last point best illustrated by scientist and philosopher Alfred Korzybski's remark "the map is not the territory"¹ as well as the Indian story of the *blind men and an elephant*,² which recognizes and underscores the diversity of interpretations based on limited sensorial information.

The former of these illustrations applies to physics most directly, because whenever we measure something, we only end up seeing a 'map' of the real thing as 'seen' through the 'eyes' of the detector we use. This in turn means that not only should we, but we must include the detector in our analysis of the other physical entity we set out to understand - in our case electromagnetic waves. Furthermore, it clears up and makes painfully obvious such so-called paradoxes as light behaving differently, i.e. as a particle or as a wave, depending on how we measure it: of course we will get different results (a different map) if we look at it differently (a different detector).

Thus, the we find it all the more important to try to understand and visualize the actual process of interaction that takes place between the detector and the measured, physical quantity (EM Waves). Depending on the actual interaction process, the measured results may vary. This paper demonstrates the results of measuring the same physical entity - two superposed EM-waves - using detectors that differ in their interaction with the same.

In addition to this, the authors want to emphasize the importance of using our main analytical tool, namely that of mathematics, very carefully, in order to capture as much of the actual physics as possible, while introducing as few unphysical artifacts as we can. We use mathematics as a means to understand, find patterns in, and quantify the physical world - i.e. we use mathematics to make a map of it. This works so remarkably well that we have begun to stop distinguishing between the two. (ex: Q: "Why can we not, in general, reverse the order of two 3-D rotations and get the same result?" - A: "Because they have, in general, a non-zero commutator.")

To reverse this counterproductive trend, as well as rid ourselves of much of the confusion and complications that have already arisen from it,^{3,4} the authors advocate a very strict interpretation of mathematical symbols and operations. For example, when considering two superposed EM-waves $\vec{E}_1(\vec{x}, t)$ and $\vec{E}_2(\vec{x}, t)$, we routinely write the resulting total field $\vec{E}(\vec{x}, t)$ as

$$\vec{E}(\vec{x}, t) = \vec{E}_1(\vec{x}, t) + \vec{E}_2(\vec{x}, t) \quad (1.1)$$

This innocent looking mathematical statement embodies many of the issues that this paper tries to address. Keeping the principle of Non-Interaction of Waves⁵ (NIW principle)* in mind, Eq.(1.1) would imply that the wave propagates through some material medium that in essence adds up the two separate waves and produces a resultant wave in some way. More specifically, the addition *operator* (+) signifies an interaction between (i.e. an operation on) the two waves, which they cannot perform by themselves.

To clarify this point some more and demonstrate that we do not simply argue semantics here, let us make the previous example a little more specific by looking at two sinusoidal waves $\vec{E}_1(t) = \cos(\omega_1 t)\hat{x}$ and $\vec{E}_2(t) = \cos(\omega_2 t)\hat{x}$. According to Eq.(1.1) and an elementary trig-identity we can then write

$$\vec{E}(t) = 2 \cos(\Omega_1 t) \cos(\Omega_2 t)\hat{x} \quad (1.2)$$

where $\Omega_1 \equiv \frac{\omega_1 - \omega_2}{2}$ and $\Omega_2 \equiv \frac{\omega_1 + \omega_2}{2}$. Eq.(1.2) represents an amplitude modulated EM-Wave with carrier frequency Ω_2 , and envelope frequency Ω_1 . This would imply that, just by superposing two EM-waves, we could amplitude modulate such a wave, which, of course, we cannot.

The problem presented here stems from our careless use of mathematics, in particular the addition operator. While multiple EM-waves do not interact on their own (NIW), we implicitly assumed that they do by writing down Eq.(1.1): the sum on the RHS *creates* a new single wave (LHS). This implies that the addition operator represents an interaction. Something needs to physically carry out the summation for us to see a resultant field in the physical volume of superposition: a detector. We can illustrate this further with an experiment very easy to carry out.

Arrange a setup of two beams of light with coplanar but not collinear Poynting vectors (i.e. such that the beams cross at some point). Without a detector present in the volume of superposition of these two beams, each beam will enter this volume and emerge unaltered on the other side, which shows that they crossed without interacting with each other. Inserting into the volume of superposition a detector of some sort will reveal fringes, whereas inserting it at any other position in any of the beams will not. Thus, we must conclude that the detector does *something* with the two independent superposed beams, which then results in the effects we see. Clearly we need to know how the detector, specifically its interaction process with EM-radiation, works in order to interpret the results.

2. DEFINITIONS

In the following discussions on different detectors and the mathematical representation between them and EM-waves we will use symbols and notation which we need to clearly define and specify in the interest of mapping and understanding the physical processes most adequately. When speaking of *detectors* we will most often mean "detecting dipoles", as we assume electric dipole-interaction to comprise the primary field-matter interaction.

Mathematically, we will characterize the dipole by a vector $\vec{\chi}$, whose magnitude corresponds to the (first-order) susceptibility, and whose direction represents the direction in which the dipole will oscillate when under the influence of the impinging wave. The vector

$$\vec{d} = (\vec{E} \cdot \vec{\chi})\hat{\chi} \quad (2.1)$$

will then represent the dipole-oscillation, and \mathcal{D} - whose mathematical representation depends on the specific type of detector/detection mechanism - the energy absorbed by the dipole.

*The NIW principle states that waves do not interact (interfere) with one another or themselves, unless in the presence of a material medium which can facilitate such an interaction.

Since we will deal with detector responses here, we can now justify the use of the addition operator when superposing the two beams: something will actually carry out the process of summation of the local fields, namely the dipole(s).

The EM-waves we will consider in the following examples will have no relative phase delay and both have the same amplitude E_0 and either the same or different frequencies. Furthermore, we will do two things to avoid unnecessary complications resulting from mathematics: firstly, when discussing the actual physical situation, we will consider two crossing *beams* of EM radiation. Secondly, when framing the problem mathematically, we will simplify things by considering two *plane waves*. This will let us focus on the important aspects of the process without introducing unnecessary mathematical difficulty. Most generally we can write down two such plane waves propagating in the x - y plane (i.e. polarized in the z -direction) as follows:

$$\begin{aligned}\vec{E}_1(\vec{x}, t) &= E_0 \cos(\vec{k}_1 \cdot \vec{x} - 2\pi\nu_1 t) \hat{z} \\ \vec{E}_2(\vec{x}, t) &= E_0 \cos(\vec{k}_2 \cdot \vec{x} - 2\pi\nu_2 t) \hat{z}\end{aligned}$$

with $\vec{x} \equiv x\hat{x} + y\hat{y}$. For the special cases considered below, we will only look at the detector response on the line defined by $y = 0$, thus $\vec{x} \equiv x\hat{x}$. Furthermore, the waves will propagate at an angle of $\phi = \pi - 2\theta$ with respect to each other (see Fig.1.a). Thus we can represent their wave vectors as $\vec{k}_1 = k_1(\cos\theta\hat{x} + \sin\theta\hat{y})$ and $\vec{k}_2 = k_2(-\cos\theta\hat{x} + \sin\theta\hat{y})$ respectively, where $k_n \equiv \frac{2\pi\nu_n}{c}$. Thus, we can represent the fields as

$$\vec{E}_1(\vec{x}, t) = E_0 \cos(k_1 x \cos\theta - 2\pi\nu_1 t) \hat{z} \tag{2.2a}$$

$$\vec{E}_2(\vec{x}, t) = E_0 \cos(k_2 x \cos\theta + 2\pi\nu_2 t) \hat{z} \tag{2.2b}$$

3. DETECTORS

When considering detecting dipoles, specifically electrons, we find that we can divide them into two groups. For one, we have the *free* and *semi-free* electrons, which directly and more or less instantaneously respond to the local field. On the other hand we have electrons in Quantum Mechanical (QM) bound states of, for example, atoms and molecules.

Free Electrons

By 'free electrons' we mean electrons that do not belong to any quantized system such as an atom or molecule. By 'Semi-free electrons' we mean free electrons that belong to some macroscopic system, for example an antenna connected to a LCR circuit. Free electrons directly respond to the local E-field amplitude in such a way that we can directly measure it. This actually allows for Fourier-synthesis to happen in nature: if multiple EM-waves co-propagate through a volume occupied by such an electron, it will respond to the instantaneous local E-field, which in this case corresponds to the local algebraic sum of these waves. An ingenious experiment confirmed this for optical fields by measuring the change in kinetic energy such fields impart on free electrons.⁶ Electrons in LCR antennas seem to represent the macroscopic analog to the quantum devices with energy bands, mentioned in the following section. One can tune these circuits such that they only respond to extremely narrow bands (or single frequencies), which effectively 'turns off' Fourier-synthesis.

Bound Electrons

When bound to an atom, molecule or other quantum device, electrons 'live' in quantized energy levels or relatively narrow energy bands. While in such a bound state, they can only absorb very specific amounts of energy at sharply defined frequencies according to the rule $\Delta E = h\nu$, where ΔE represents the energy difference between the initial and final state, and ν the frequency of the absorbed radiation. The reverse holds true, as well: when transitioning from higher to lower states of energy, these electrons emit energy packets described by the same rule.

The concept of discrete, particle-like photons seems to originate from this behaviour. Such quantum devices, as opposed to free electrons, need some amount of time to sense their 'compatibility'⁷ with the frequency of the impinging radiation, and cannot, due to their narrow frequency range, perform Fourier-synthesis in general. Quantum devices with energy bands relax the latter somewhat. Since such detectors only produce measurable signals upon absorption of energy, they restrict the information we can extract about the sensed wave to the square of the amplitude, i.e. the energy.

In the following sections we will look at a number of different detector models, and what each of them would 'see' within the superposition volume of two crossing light beams.

4. AMPLITUDE DETECTION

Now, let us suppose that we have a detector that can directly and instantaneously respond to the amplitude of the electric field. We can represent this response mathematically by Eq.(4.1).

$$\mathcal{D} = |\vec{d}| = \chi|\vec{E}_1| + \chi|\vec{E}_2| \quad (4.1)$$

Same Frequencies

If using Eq.(2.2) and both beams have the same frequency ν_0 this becomes

$$\begin{aligned} \mathcal{D} &= \chi E_0 \cos(k_0 x \cos \theta - 2\pi\nu_0 t) + \chi E_0 \cos(k_0 x \cos \theta + 2\pi\nu_0 t) \\ &= 2\chi E_0 \cos(k_0 x \cos \theta) \cos(2\pi\nu_0 t) \\ &= 2\chi E_0 \cos\left(2\pi\nu_0 \frac{x}{c} \cos \theta\right) \cos(2\pi\nu_0 t) \end{aligned} \quad (4.2)$$

where $k_0 \equiv \frac{2\pi\nu_0}{c}$. In this model we implicitly assumed that the detecting dipoles possess the freedom to align themselves in the direction of the electric field. Things would look somewhat different in the case of a polarized detector-medium where this freedom does not exist. For example, if the medium has its polarization perpendicular to the polarization of the EM-wave, the detecting dipoles would register nothing at all: it would *look* to us as if no wave hit the detector, even though it certainly did. We will not consider this situation here in any more detail.

Eq.(4.2) states that the detector responds to the stimulation of the two independent waves simultaneously, effectively summing the two signals. The addition operator serves as mathematical representation of this process. However, we need to point out that there exist almost⁶ no known detectors for EM-waves in the optical regime that behave in this manner. LCR circuits involved in the detection of radio waves, on the other hand, *do* present to us a measurable transformation proportional to the sum of the amplitude stimulation.

Fig.1 shows the result of instantaneous amplitude detection visually. Fig.1.c shows the detected signal at one instant in time. As time advances, this waveform will remain stationary (standing wave), while the amplitude oscillates between zero and $2\chi E_0 \cos\left(2\pi\nu_0 \frac{x}{c} \cos \theta\right)$. (Fig.1.d)

Different Frequencies

If using Eq.(2.2) and both beams have *different* frequencies, the math does not simplify as nicely. Let us call the two frequencies ν_1 and ν_2 - now we can write the detector's response as:

$$\begin{aligned} \mathcal{D} &= \chi E_0 \cos(k_1 x \cos \theta - 2\pi\nu_1 t) + \chi E_0 \cos\left(\frac{k_2}{2} x \cos \theta + 2\pi\nu_2 t\right) \\ &= \chi E_0 \cos\left[2\pi\nu_1\left(\frac{x}{c} \cos \theta - t\right)\right] + \chi E_0 \cos\left[2\pi\nu_2\left(\frac{x}{c} \cos \theta + t\right)\right] \end{aligned} \quad (4.3)$$

Where we used $k_n = \frac{2\pi\nu_n}{c}$. Fig.2 demonstrates the detector response in this case. Fig.2.d shows the time evolution of the amplitude-fringes, which does not exhibit any obvious patterns. If we integrated the waveform in time over about 1 period (not shown), we would see the pattern steadily moving to the left.

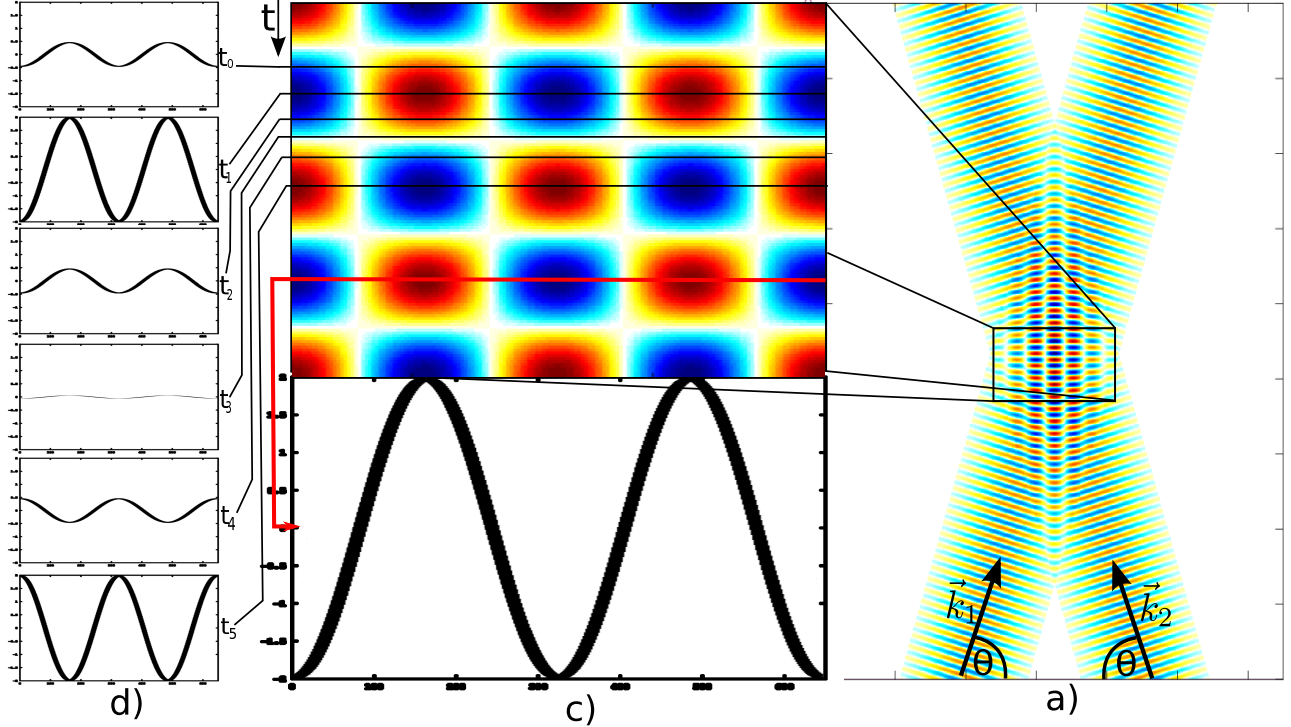


Figure 1: a) Map of summed amplitudes of two coplanar, non collinear EM-beams with equal frequencies ($\nu_2 = \nu_1$). b) Superposition region of the two beams. The warm colors indicate positive amplitude values (peaks) and the cold colors negative values (troughs). c) Cross section of the superposition region at one particular time. d) Successive snapshots of the instantaneous amplitude, taken at the times indicated, where $t_0 < t_1 < t_2 < t_3 < t_4 < t_5$

5. INSTANTANEOUS ENERGY DETECTION

Often, especially in the optical domain, we cannot measure the amplitude oscillation directly - the detector only gives us the energy absorbed, i.e. a quantity proportional to the square of the amplitude. This generally applies to optical frequencies and higher, such as x-rays, γ -rays, etc. Now let us look at the response of a detector, which can respond only to the instantaneous local E-field energy, and which we can represent mathematically by Eq.(5.1).

$$\mathcal{D} = (|\vec{d}|)^2 = (\chi|\vec{E}_1| + \chi|\vec{E}_2|)^2 \quad (5.1)$$

Once again we will look at the two cases of equal and different frequencies between the two beams.

Same Frequency

$$\mathcal{D} = 4\chi^2 E_0^2 \cos^2(2\pi\nu_0 \frac{x}{c} \cos \theta) \cos^2(2\pi\nu_0 t) \quad (5.2)$$

The dipoles respond to each of the beams separately, and absorb energy proportional to the squared sum of the individual dipole stimulations (Fig.3). Note that the fringes in Fig.3.d, although stationary in the spatial sense, still have a dynamical component to them, as their height oscillates between the maximum ($4\chi^2 E_0^2 \cos^2(2\pi\nu_0 \frac{x}{c} \cos \theta)$) and zero with time.

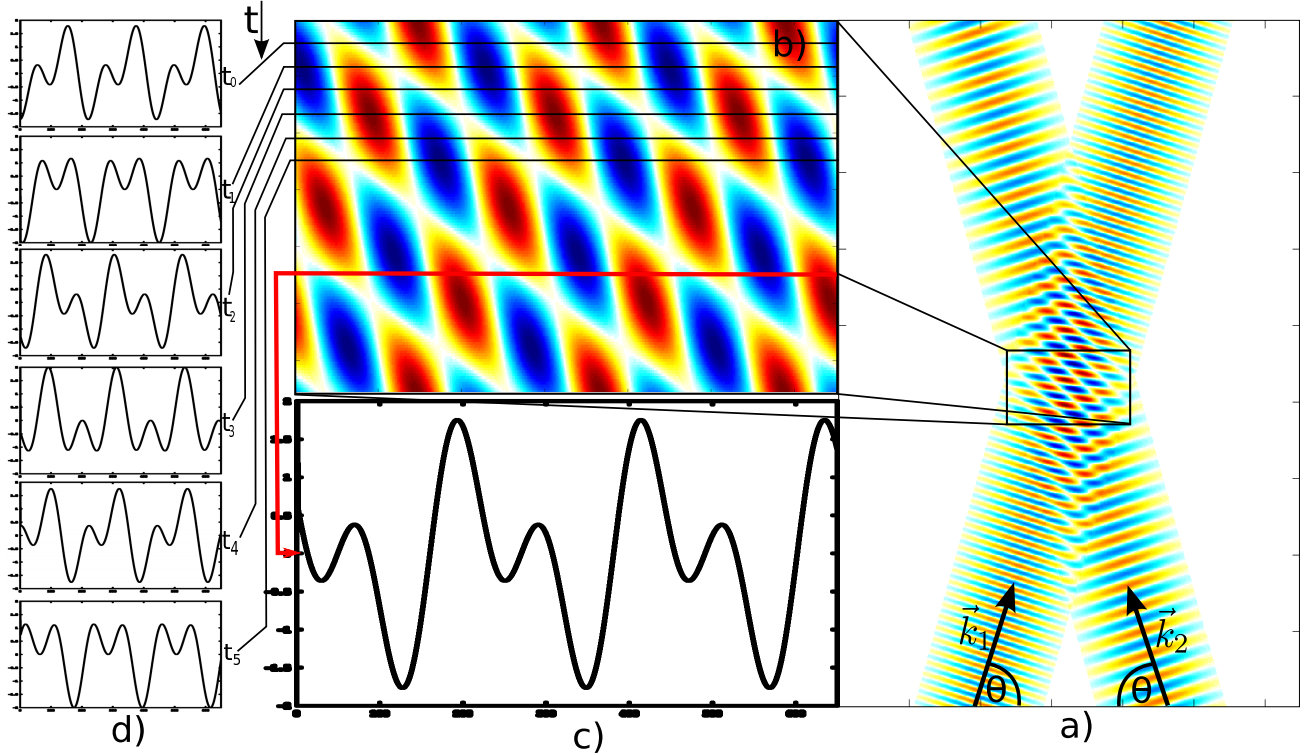


Figure 2: a) Map of summed amplitudes of two coplanar, non collinear EM-beams with different frequencies ($\nu_2 = \frac{1}{2}\nu_1$). b) Superposition region of the two beams. The warm colors indicate positive amplitude values (peaks) and the cold colors negative values (troughs). c) Cross section of the superposition region at one particular time. d) Successive snapshots of the instantaneous amplitude, taken at the times indicated, where $t_0 < t_1 < t_2 < t_3 < t_4 < t_5$

Different Frequencies

$$\begin{aligned}
 \mathcal{D} = & \chi^2 E_0^2 \cos^2 \left[2\pi\nu_1 \left(\frac{x}{c} \cos \theta - t \right) \right] + \chi^2 E_0^2 \cos^2 \left[2\pi\nu_2 \left(\frac{x}{c} \cos \theta + t \right) \right] \\
 & + 2\chi^2 E_0^2 \cos^2 \left[2\pi\nu_1 \left(\frac{x}{c} \cos \theta - t \right) \right] \cos^2 \left[2\pi\nu_2 \left(\frac{x}{c} \cos \theta + t \right) \right]
 \end{aligned} \tag{5.3}$$

Just as in the case of the instantaneous amplitudes, the instantaneous energy fringes have a highly dynamic character as exemplified by Fig.4.d.

6. TIME-INTEGRATED ENERGY DETECTION

Unlike the detector we just considered, real detectors in the optical domain do not detect the instantaneous energy. It appears that all such detectors have an intrinsic time-constant, meaning they absorb energy over a given period of time, depending on the particular detector. We also need to point out another important aspect of these detectors, namely that they have their basis in Quantum Mechanical (QM) transitions - electronic or otherwise. This in turn makes them very different from detectors used to detect, for example, Radio Frequency (RF) EM-waves. Whereas RF detectors absorb energy continuously, optical frequencies induce *transitions* in QM detectors that require sharply defined amounts of energy. If the conditions ($E = h\nu$) match as described in Section 3 these detectors will respond, otherwise we will register no response at all. Thus, these devices *appear* to detect discrete photons (quanta of light), regardless of whether an actual EM-wave *packet* triggered the detector response, or a section of a more continuous wave with the same frequency.

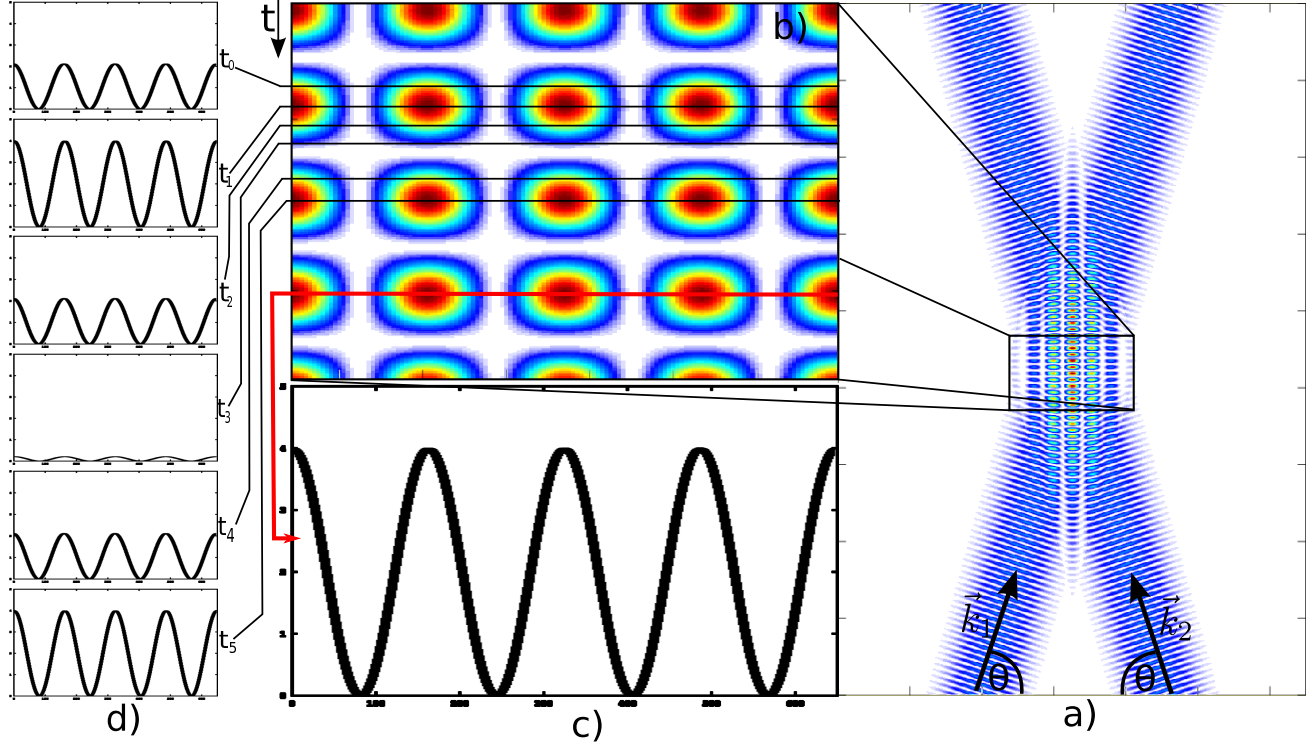


Figure 3: a) Map of local total energy of two coplanar, non collinear EM-beams with equal frequencies ($\nu_2 = \nu_1$). b) Superposition region of the two beams. The warm colors indicate larger energy values (peaks) and the cold colors small values. c) Cross section of the superposition region at one particular time. d) Successive snapshots of the instantaneous energy, taken at the times indicated, where $t_0 < t_1 < t_2 < t_3 < t_4 < t_5$

In order for the detector to determine its compatibility⁷ with the impinging wave (i.e. to determine whether it has the right frequency), and to absorb the required amount of energy, the detector effectively time-integrates over a few periods of the wave, which we mathematically realize by time-integrating Eq.(5.1). In terms of math we will from now on focus on one particular point in space ($x = 0$ for convenience), as opposed to a whole section of the x-axis, as this will simplify the discussion, especially of the general (different frequencies) case, tremendously. We now use Eq.(6.1) to represent the detector response.

$$\mathcal{D} = \int_0^T (|\vec{d}|)^2 dt = \int_0^T (\chi|\vec{E}_1| + \chi|\vec{E}_2|)^2 dt \quad (6.1)$$

Same Frequency

$$\begin{aligned} \mathcal{D} &= \int_0^T \chi^2 E_0^2 \cos^2 2\pi\nu_0 t + \chi^2 E_0^2 \cos^2 2\pi\nu_0 t + 2\chi^2 E_0^2 \cos^2 2\pi\nu_0 t dt \\ &= 2\chi^2 E_0^2 \end{aligned} \quad (6.2)$$

Fig.5 shows that when the detector time-integrates over about 1.5 periods of oscillation, the fringes it creates in the area of physical superposition of the beams appear stationary in space as well as in time. Even though the beams themselves have not changed - the two beams propagate independently - this particular detector shows us a completely time independent, i.e. steady, fringe pattern. Once again, this does not represent any property of the EM-waves themselves, but depends entirely on the detector. It turns out that once T has a large enough value (a few periods of oscillation - depending on the angle between the Poynting vectors of the beams), this detector time constant has practically no more influence on this property of the fringes.

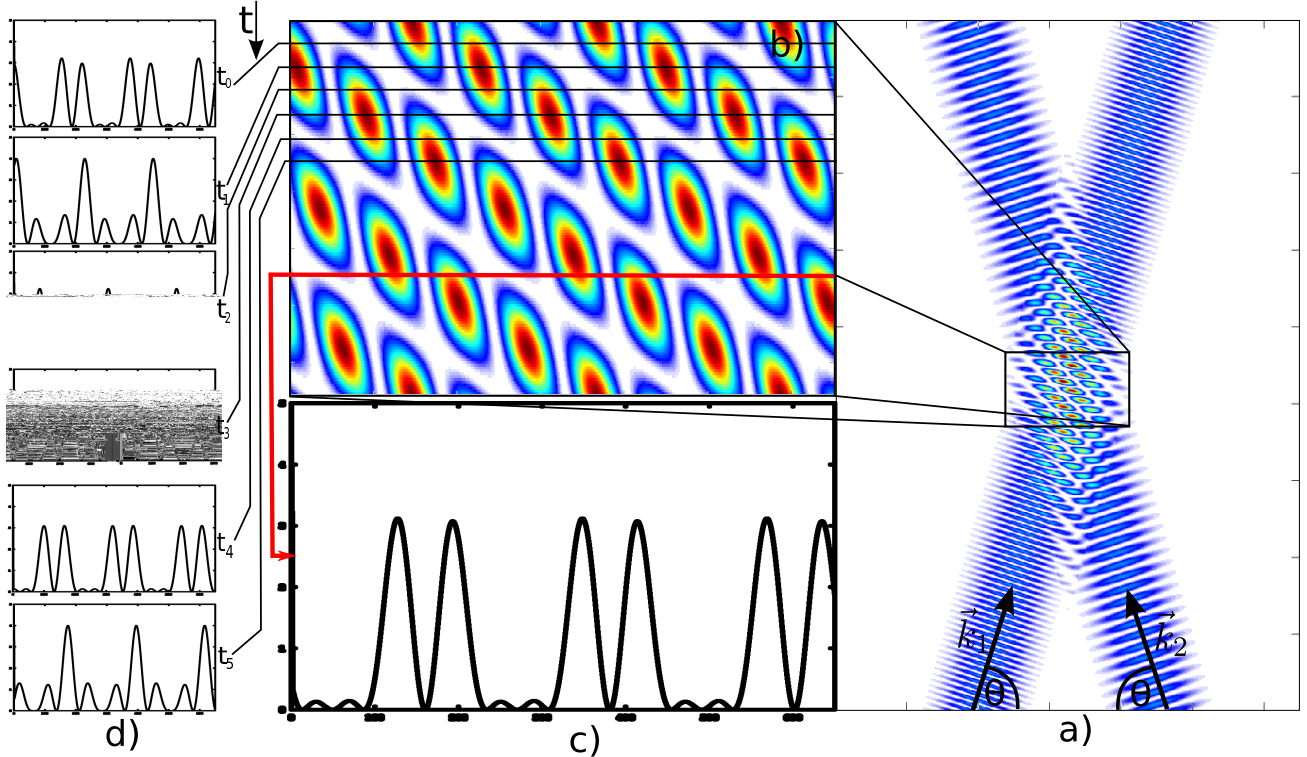


Figure 4: a) Map of local total energy of two coplanar, non collinear EM-beams with different frequencies ($\nu_2 = \frac{1}{2}\nu_1$). b) Superposition region of the two beams. The warm colors indicate larger energy values (peaks) and the cold colors small values. c) Cross section of the superposition region at one particular time. d) Successive snapshots of the instantaneous energy, taken at the times indicated, where $t_0 < t_1 < t_2 < t_3 < t_4 < t_5$

Different Frequencies

$$\mathcal{D} = \int_0^T \chi^2 E_0^2 \cos^2 2\pi\nu_1 t + \chi^2 E_0^2 \cos^2 2\pi\nu_2 t + 2\chi^2 E_0^2 \cos(2\pi\nu_1 t) \cos(2\pi\nu_2 t) dt \quad (6.3)$$

Whereas in the equal frequencies case the value of T does not make much of a difference, it does matter when the frequencies differ. Clearly the outcome of the integral Eq.(6.3), and with that the result the detector will give us, depends crucially on this time span. This means that the integration time depends on the particular detector we use and hence represents a property of said detector - its inherent time constant. Unlike in the cases of direct amplitude or energy detection, when incorporating time integration, the detection results will depend quite significantly on the detailed properties of the detector. As a specific example of this we mention the case of a multi longitudinal mode CW He-Ne laser beam. If received by a slow detector, we find a steady CW intensity. The same beam, when received by a detector with a time-constant faster than the difference frequency of the modes, will give an oscillatory current that will contain all possible difference frequencies between the various modes.

7. DISCUSSION

As we have demonstrated, one and the same physical situation will look very different through the eyes of different detectors. Furthermore, We have insisted on using entirely *real* notation, as opposed to the customary complex representation of EM-waves. In the previous section we have pointed out the important fact that most every QM detector inherently time-integrates in the detection process. Apart from introducing an un-physical extra term into our mathematical representation, the complex representation hides the time integration process, and

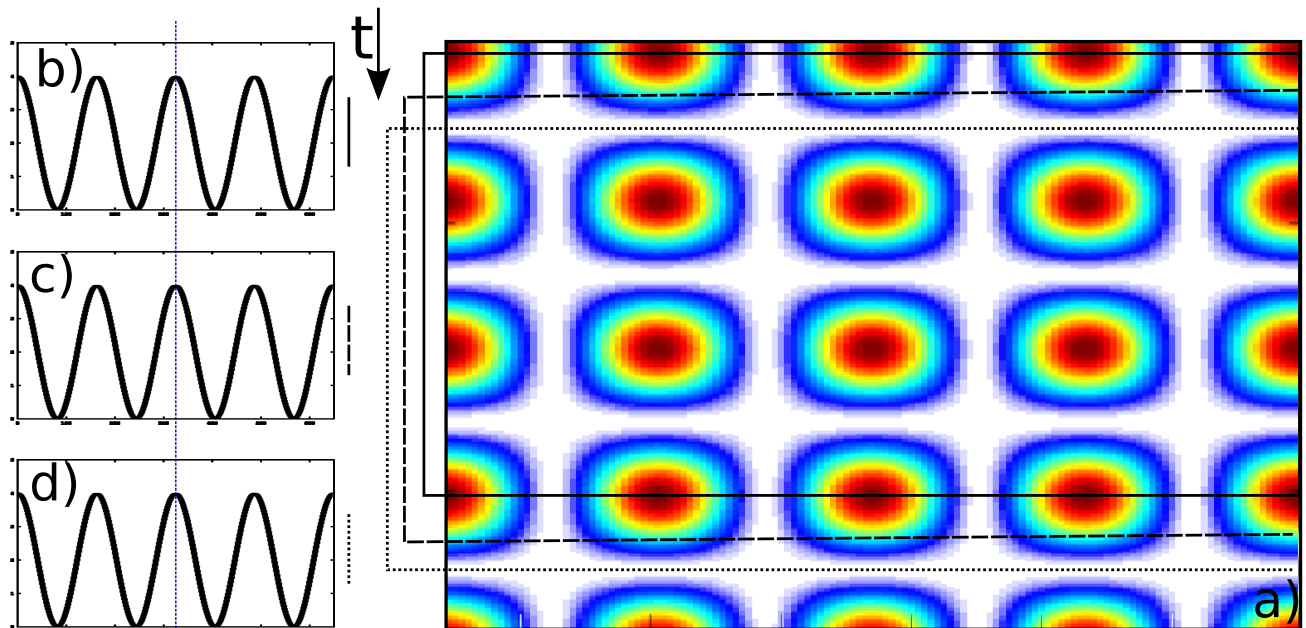


Figure 5: Time-integrated energy detection with equal frequencies. a) Superposition region of the two beams as seen by a non-integrating detector. Red regions correspond to maximum values, while white corresponds to zero. b), c), d) The results of time-integration of equal length T , starting at different points in time, as indicated. The result does not depend on the exact point in time at which we begin the measurement.

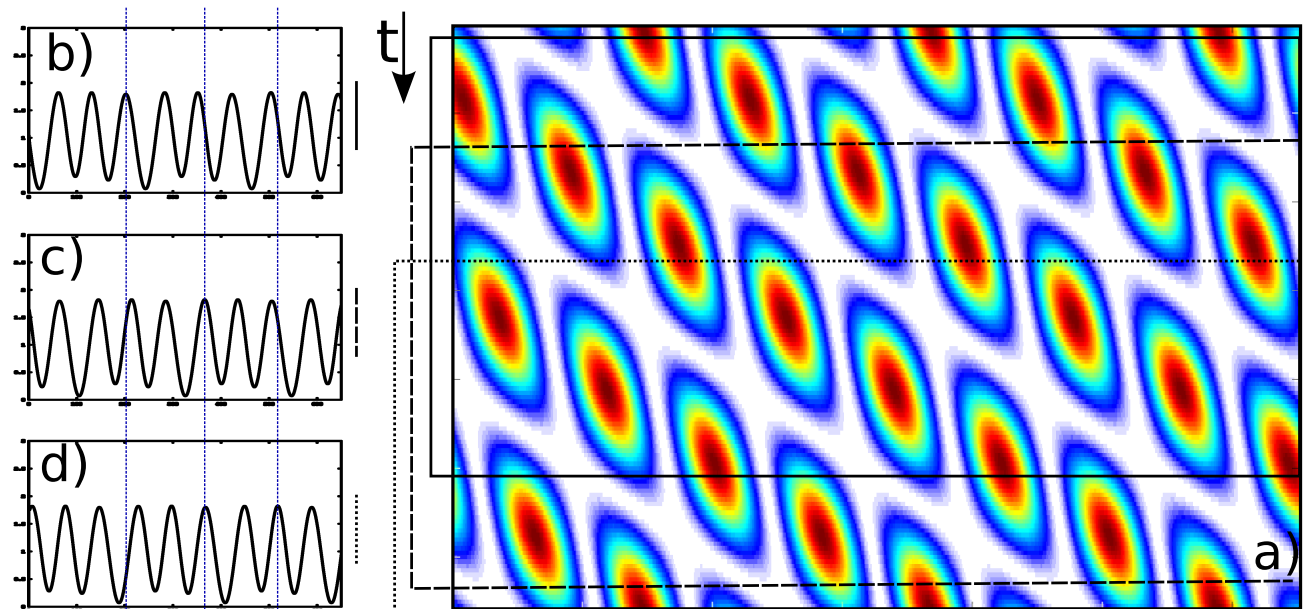


Figure 6: Time-integrated energy detection with different frequencies ($\nu_2 = \frac{1}{2}\nu_1$). a) Superposition region of the two beams as seen by a non-integrating detector. Red regions correspond to maximum values, while white corresponds to zero. b), c), d) The results of time-integration of equal length T , starting at different points in time, as indicated. In this case the phase of the result does depend on the initial time, while the maximum amplitude does not.

thus part of the actual physics, and we should thus avoid it[†]. Consider the two beams from before, with both having the same frequency. Again we will only look at position $x = 0$ for clarities sake. According to Eq.(6.2) the detector will absorb

$$D = 2\chi^2 E_0^2$$

Now, instead of using real representation, let us represent our beams by the following two *complex* equations:

$$\begin{aligned}\vec{E}_1 &= E_0 e^{2\pi i \nu_1 t} \hat{z} \\ \vec{E}_2 &= E_0 e^{2\pi i \nu_2 t} \hat{z}\end{aligned}\tag{7.1}$$

where $\nu_1 = \nu_2$. We get the energy detected by calculating the square-modulus of the now complex dipole oscillation as follows:

$$\mathcal{D} = d^* d = |d|^2 = 4\chi^2 E_0^2$$

Save for a factor of two, the results coincide, which implies that the complex square modulus in a sense includes a time-integration over two periods! While this seems reasonably convenient and works in many cases, the problem arises when our detector's time constant deviates substantially from this value. Using real representation in conjunction with time-integration over a suitable time period represents a much more general way of describing the physics while also staying closer to the interaction processes involved.

To further illustrate the shortcomings of the complex representation, let us look at optical heterodyne. Using Eq.(7.1) again but leaving the two frequencies different, we calculate the following detector response:

$$\mathcal{D} = |d|^2 = 2\chi^2 E_0^2 + 2\chi^2 E_0^2 \cos [2\pi(\nu_2 - \nu_1)t]$$

This has practically eliminated the sum frequency, which we know from RF heterodyne can get produced, and leaves us only with the difference frequency. Again, the complex notation captures only part of the physical process, and if the QM device we use has a small enough time constant, the complex representation will fail entirely.

These two examples illustrate the importance of using mathematics that maps the physical processes most closely. Relying too much on convenient mathematical representations that hide part of the physics only results in confusion later on. As discussed earlier, the Fourier Transform formalism serves as another instance of this. Just because a mathematical prescription works in some or most cases, does not mean that it represents a good mapping between our logic and the process of nature we try to summarize with it. A *good* map represents the physical process in such a way that we can visualize it, in our minds, which in turn will enable us to, for example, develop new technologies more easily.

[†]Nevertheless, this mathematical short-cut may provide considerable computational and notational advantages. We just need to remain keenly aware of its limitations and not lose sight of the various detailed steps detectors undergo before providing us with measurable data.

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