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CAUSALITY AND CLASSICAL INTERFERENCE AND DIFFRACTION PHENOMENA

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SUMARIO

Si el principio de superposición lineal es una realidad física, el efecto de superposición no puede ser observado hasta que las causas (señales) hayan llegado al espacio y punto-temporal bajo consideración. Entonces, los patrones de interferencia de haces múltiples debido a los interferómetros Fabry-Perot y a las rejillas en cualquier tipo de iluminación no-estacionaria deben desarrollarse en el tiempo en lugar de producir un patrón estacionario instantáneamente; la razón es que los haces múltiples sufren retrazos de fase que corresponden a retrazos en el tiempo. En base a esto, se analiza el comportamiento temporal de los patrones de interferencia y el poder de resolución de las rejillas, encontrando posibles nuevas aplicaciones como modulación y generación de pulsos, y además como dispositivos para medir el ancho del pulso. El presente análisis utiliza el principio de Huygens-Fresnel en el espacio real en vez de las técnicas del espacio transformado de Fourier. El análisis implica que el pulso de picosegundos no puede utilizar ventajosamente el poder de resolución de rejillas grandes. Además, en el apéndice se presenta el tratamiento para calcular el patrón de difracción temporal producido por una sola rendija.

ABSTRACT

If the principle of linear superposition is a physical reality, then the effect of superposition cannot be observed until the causes (signals) have arrived at the space and time point under consideration. Then multiple beam interference patterns due to Fabry-Perot interferometers and gratings under any non-stationary illumination should develop with time instead of reaching the steady-state pattern instantaneously, because the multiple beams have phase delays corresponding to time delays with respect to each other. On this basis, we have analyzed the temporal development of interference patterns and resolving power due to gratings and find possible new applications of gratings as pulse shapers, pulse generators, or pulse width measuring device. Our analysis follows the Huygens-Fresnel principle in real space rather than the techniques of the Fourier transformed space. The analysis implies that a picosecond light pulse cannot exploit the high resolution of large gratings. Time diffraction due to a single slit is also developed in the appendix.

1. *Introduction*

Classical causality implies that interference patterns produced by any conventional interferometer are due to the real physical superposition of more than one physical signal carrying different information such as phase and frequency. This is easy to demonstrate experimentally with interferometers. A two-beam Michelson interferometer set for dissonance (zero transmission toward the observer) with a path difference d produces a short pulse of duration d/c when a continuous wave (cw) beam is suddenly pulsed, due to the delay in arrival of the second beam compared to the first. Analysis and experimental work along these lines have been reported by Szoke et al (1972) and Milam et al (1974). Because Fabry-Perot interferometer is a multiple beam type, the situation is a bit more complicated. But Bradley et al (1964, 1968) reported that the interference pattern develops with time (see also Stoner 1966). Then Kastler (1974) analyzed the propagation of a single square pulse through a Fabry-Perot. In the same year Roychoudhuri (1974a) also reported a generalized analysis of the temporal response of Fabry-Perots to both single pulse and series of pulses of various widths and separations; the limits of the conventional spectrometric analysis of the Fabry-Perot fringes under non-stationary situations were also discussed by this author (Roychoudhuri 1975a). There it was pointed out that the production of dispersion (spectral separation) through interference requires the decomposition of the original wavetrain into many and then their physical superposition after the introduction of a sequential delay (see also Roychoudhuri 1976a). Further analysis and experimental works on Fabry-Perot using ultrashort pulses have also been reported (Martin and Milam 1976, Milam and Martin 1976, Martin 1977).

The importance of short pulse phenomenon is not limited to the field of basic physics for understanding the nature of radiation. Applied fields like laser-fusion requires a properly tailored pulse for inducing efficient implosion. Some solutions to this problem have been obtained by stacking a series of pulses with the help of a set of mirrors and beam-splitters (Soures et al 1974) or a pair of Fabry-Perot type etalons (Martin and Milam 1976, Thomas and Siebert 1976). The concept of real physical superposition of a train of pulses with suitable delays is the basis of all these articles. The pulse train can be obtained directly from a laser (Roychoudhuri 1977) or by replicating a single incident pulse (Martin and Milam 1976, Thomas and Siebert 1976). Some of these articles have recognized that the real physical interference of pulses gives rise to the effect of dispersion (spectral separation). But one can encounter articles (Duguay and Hansen 1969, Treacy 1969) in the literature using the dispersive properties of Fabry-Perots and gratings for short pulses without referring to the temporal limitations.

The main objective of this paper (Roychoudhuri 1975b, 1976b) is to analyze the effect of gratings to short pulses using this concept of real physical superposition that is established in classical interferometry as cited above. To avoid confusion, we point out that we shall treat our light pulses (with well defined space and time extension) as purely classical waves (with sufficient energy for conventional detection). They are not considered here as either quantum mechanical wave packets or photons. It is now generally accepted that a very large number of optical phenomena (including photoelectric effect) can be explained by semiclassical and neoclassical radiation theories without using quantum electrodynamics (Scully and Sargent 1972, Mandel 1976). Regarding the capability of a high resolution grating in separating the component frequencies of an incident radiation, many scientists assume an ad hoc hypothesis that gratings send instantaneously the various frequencies into the characteristic directions Θ given by $d \sin \Theta = m\lambda = mc/\nu$ where d is the grating period. For example, see the articles by Treacy (1969), Landé (1975) and also the reply to my questions at the end of the paper by Busch et al (1976). Such an ad hoc hypothesis does not have the support of either experimental work or any basic principles of physics. Besides, the hypothesis of an instantaneous interaction of an extended object like a grating with a space and time extended radiation pulse violates causality (Roychoudhuri 1975c, Roychoudhuri et al 1976). It is to be noted that the principle of causality is accepted in all branches of Physics. Only in the branch of particle physics does one discusses the possibility of a violation of "causality in the weaker sense" (within the uncertainly limit $\delta E \delta t \geqslant 1$) and even that is a controversial issue (Heisenberg 1959, Jammer 1974).

A Fabry-Perot replicates the incident wave into a train of delayed waves by amplitude division, so does a grating by wavefront division through each of its slits (or steps). An elementary way of demonstrating the equivalency of a Fabry-Perot and a grating as producers of multiple beam interference has been reported before (Roychoudhuri 1974b). To maintain causality, we assume that all the grating wavefronts propagate with different delays in different directions and for frequency separation, the incident wave must be long enough that there is real physical superposition of these delayed replicated waves. So, in the extreme case of an echelon grating having a step-delay larger than the width of an incident pulse, one should obtain a train of N independent pulses identical to the incident one, instead of an instantaneous separation of the component frequencies in different directions. Such a concept is not at all new in classical optics. For recent experimental applications of this technique see references (Busch et al 1976, Top et al 1971). The reverse idea of superposing a train of pulses into a single one for application to the problem of laser fusion has been reported elsewhere (Roychoudhuri 1977). The general problem of time evolution of short pulses due to diffraction is now well recognized (Lugovoi 1975, Evans 1976, Caufield and Hirschfeld 1977). Lugovoi (1975) has given an analysis of a method of estimating the width of an ultrashort pulse using the focusing and diffraction properties of a lens and a zone plate. Further treatment on the time broadening of a short pulse due to diffraction by lenses has been given by Evans (1976) and Caufield and Hirschfeld (1977).

In order to explain the finite response time of a grating under pulsed illumination, we have used the Huygens-Fresnel principle that is simplest yet forms the main foundation of all theories of wave propagation. To eliminate confusion, we have avoided using Fourier integral decomposition of a time-pulse into many monochromatic radiations. The carrier frequency of the pulse is the only real physical frequency. We should mention that while Hopkins (1967) and Froehly et al (1973) (see also Viénot et al 1977) have used the concept of Fourier transform under related contexts, they have not discussed the point that the very high dispersive power of a Fabry-Perot or a grating cannot be imparted to picosecond light pulses. But this has been briefly done by Caulfield et al (1976). For ultrashort light pulses we shall see that the instrumental response function for a grating or a Fabry-Perot is, in general, a time varying one rather than being steady. While analyzing short pulse phenomena, it is important to remember that the technique of Fourier decomposition of a time pulse is not a physical principle but a mathematical theorem. Further, Fourier integral is a non-causal integral. So its misuse could give incorrect results in physics however elegant the analysis may appear (Roychoudhuri 1976a, 1976b, Rouchoudhuri and Calixto 1977) (see also the appendix).

2. *Huygens-Fresnel Wavelet Approach To Multiple Beam Interference Due To Gratings*

We first explain the origin of the time delay between the multiple beams produced by a plane grating at a non-zero diffracted order from the elementary Huygens-Fresnel principle (Born and Wolf 1975). The same result can also be obtained with mathematical rigour using any of the diffraction theories. Actually the concept that different parts of a diffracted wavefront from a grating should arrive at the plane of detection with different delays, has been used under various situations (Schuster 1894, Feynman et al 1966). The time diffraction pattern due to a single slit is developed in the Appendix. Here we shall consider a grating of very narrow slits with periodicity d (Fig. 1). It is placed at the front focal plane of the lens L. X is the observation plane placed at the rear focal plane of the

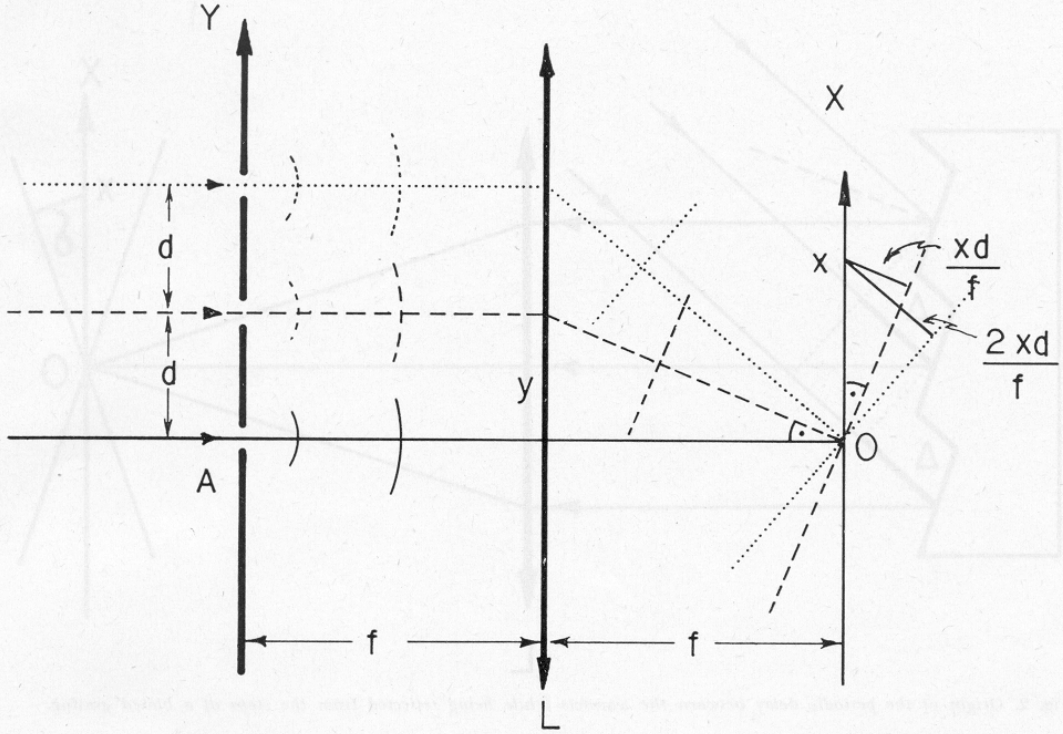

Fig. 1. *Periodic delay of the Huygens-Fresnel wavelets in arriving at the observation plane from the grating plane.*

same lens. Each slit of the grating gives rise to a Huygens-Fresnel spherical wavelet which becomes a plane wavelet after passing through the lens. This is the standard arrangement to obtain an exact Fraunhofer diffraction pattern. The wavelet, originating at the axial point A, passes through the lens parallel to the X-plane. All off-axis wavelets, although passing through the axial point O in the X-plane, arrive at angles given by,

$$\Theta_n \simeq n(d/f), \tag{1}$$

where d is the grating period, f is the focal length of the lens and n denotes the wavelet originating from the n-th slit. Thus the delay for the n-th wavelet at a point x of the X-plane is,

$$\delta_n = n(xd/f). \tag{2}$$

The periodic path difference between any pair of consecutive wavelet is,

$$\delta = xd/f, \tag{3}$$

and the periodic time-delay is,

$$\tau = xd/fc. \tag{4}$$

The order of interference is given by,

$$m = \delta/\lambda_0 = \nu_0\tau. \tag{5}$$

We assume that the pulse is obtained from an ideal ultrafast switching device in front of a stabilized single-mode (ν_0) cw laser. This carrier frequency ν_0 is the physical frequency of the radiation. We assume the width of the slit to be very small compared to the focal length of the lens. The distance x on the observation plane is much smaller than the width of any one of the single-slit patterns. Under these conditions the inclination factor of any rigorous diffraction theory reduces to unity.

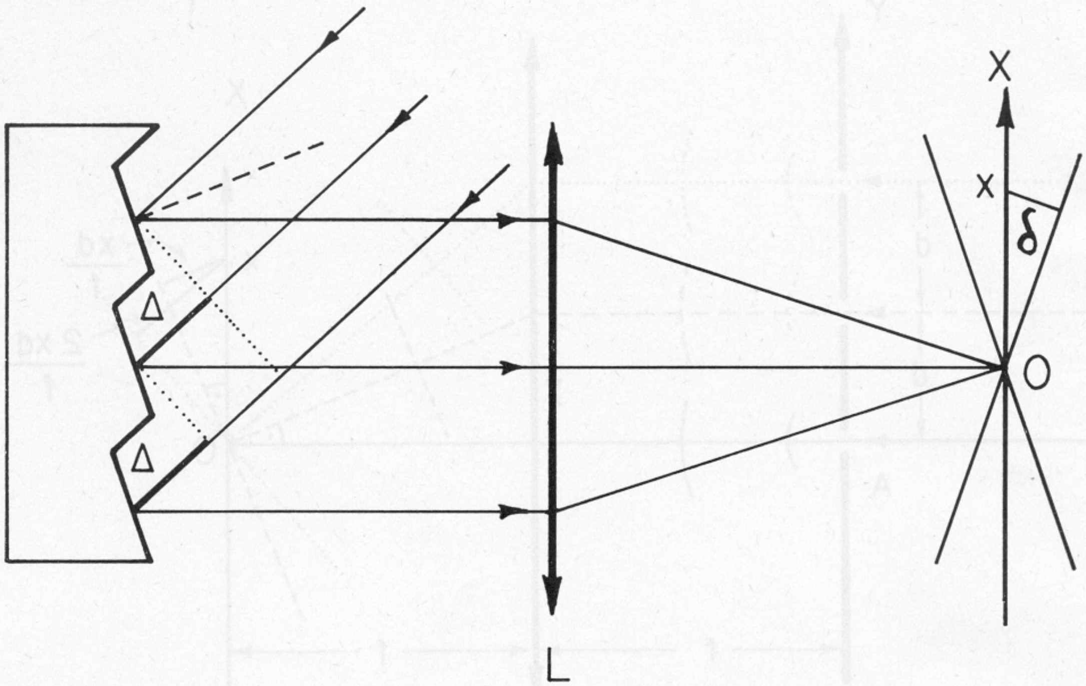

Fig. 2. *Origin of the periodic delay between the wavelets while being reflected from the steps of a blazed grating.*

We consider large step echelon type gratings because they have high resolution and high light gathering power. Since these gratings can produce a large path delay, they are more powerful instruments for pulse-shaping and pulse-generating than plane gratings. Figure 2 shows a small section of such a grating. An incident wavefront is split into a series of wavefronts by reflection from the grating steps. After reflection the path difference between any two consecutive wavefronts is Δ which can be computed from the relevant geometrical data. At the axial focal point O of the lens L we have the blazed order m,

$$m = \Delta/\lambda_0. \tag{6}$$

Correspondingly, the common-difference time delay between the series of wavefronts is,

$$\tau_m = \Delta/c = m/\nu_0. \tag{7}$$

But, away from the axial point, there is an extra path difference due to the tilt of the wavefronts produced by the lens L which is given by δ and can be computed very similarly to Eq. 3. The total order of interference is,

$$m = (\Delta \pm \delta)/\lambda_0, \tag{8}$$

and the delay-time is,

$$\tau_m = (\Delta + \delta)/c = m/\nu_0. \tag{9}$$

An incident time pulse $V(t)$ cos $2\pi\nu_0 t$ spatially extending over the entire grating plane will produce a series of similar pulses which will arrive at the observation plane with delays $n\tau_m$ where n is the slit number. The corresponding amplitude response is,

$$g(t, \tau_m) = \sum_{n=0}^{N-1} f(t - n\tau_m) \cos [2\pi_0 (t - n_m\tau)]. \tag{10}$$

For convenience, we are assuming that the arbitrary source phase is zero and the moment of arrival of the first pulse at the observation point is t. Following causality, if the width of $f(t)$ is $n\tau_m$, the sum extends over n terms $(n \leqslant N)$. Further, since the series of N beams are temporally delayed from each other, they do not arrive at the observational plane at the same instant. Thus grating patterns, like those of Fabry-Perot and Michelson interferometers, also envolve with time. But, when the field $f(t)$ has reached the steady-state (and extends from $+\infty$ to $-\infty$) and assuming the amplitude due to each slit to be unity, then the steady-state grating irradiance, with long time average for conventional slow detectors, is

$$|g_s|^2 = \frac{1}{T} \int_0^T \left| \sum_{n=0}^{N-1} \cos \left[2\pi\nu_0 (t - n\tau_m)\right] \right|^2 dt$$

$$= \frac{1}{2} \frac{\sin^2 \pi\nu_0 N\tau_m}{\sin^2 \pi\nu_0 \tau_m} \qquad (11)$$

Later we will ignore the constant factor 1/2 just for convenience of writing. The same Eq. 11 also gives the temporally evolving irradiance for a transient illumination where $f(t)$ is wide enough that several pulses are physically superposed; but N must be replaced by the running integer n where the running time is $t = n\tau_m$ and τ_m is large compared to one optical cycle $1/\nu_0$. Observation of the pattern at an integral order $m = \nu_0\tau_m$, the time evolving irradiance without normalization, is

$$|g(t = n\tau_m)|^2 = n^2 \qquad (12)$$

It is to be noted that the time average of Eq. 11 would not be valid if the original pulse contained one or two cycles of oscillation. Under this condition, the wave should be represented by a real function rather than a complex one and one can do at best the time integration to find the total energy rather than time-averaging to find the energy per unit time. Further, the arbitrary source phase factor will be very important. In short, the concept of coherence, spectroscopy and diffraction will have to be redefined for extremely short pulses

2.1 HUYGENS-FRESNEL WAVELETS IN THE NEAR FIELD OF A HIGH FREQUENCY GRATING

In normal spectroscopic works one usually records the Fraunhofer diffraction pattern at the Fourier transform (rear focal) plane of a lens. This is what we have done above. But it is instructive to look at the near field for a high frequency grating (Shulman 1970). When the separation d between two consecutive grating grooves is so small that it subtends only several degrees at a point within the near field (for the whole grating), the Huygens' secondary wavelets from the nearby independent grooves are spatially superposed (Fig. 3). Such real physical superposition of the appropriate sets of secondary wavelets gives rise to the various diffracted wavefronts that propagate ultimately as independent diffraction orders. For low frequency gratings like echelons, the wavelets from the nearby steps cannot interfere appreciably in the near field and the Fraunhofer pattern can be generated in the laboratory by superposing them at the rear focal plane of a lens. In Fig. 3 we are considering a small section (18 grooves) of a high frequency grating illuminated by a rectangular pulse of width δt containing 17 complete optical cycles of a radiation of wavelength λ_1. We are assuming that the pulse is produced by an ideal chopper from a stabilized continuous wave laser running in two independent longitudinal modes ($\lambda_1 : \lambda_2 = 5{:}6$). So the pulse will contain a little over 14 optical cycles of λ_2 radiation.

For convenience of geometric analysis we assume that the grating grooves are ideal lines. In the absence of the grating the pulse would have been occupying at some moment the space between the lines AB and CD where $AC = BD = 17\lambda_1$. But the presence of the 18 line grating (with grating constant $d = 3\lambda_1$) has caused the pulse to be composed of 18 secondary spherical wavelets at every plane between AB and CD. We are showing only a few specific sets out of all these wavelets. AD and EF are showing the formation of first and second order diffracted wavefronts for λ_1. These wavelets are drawn with radii $JA + n\lambda_1$ where n runs from 0 to 17 for the consecutive grating lines. Similarly, BG and HI are showing the formation of first and second order wavefronts for λ_2. It is to be noted that due to the limited duration of the incident pulse, only the first order diffracted wavefront due to radiation λ_1 has 18 wavelets from all the 18 lines. So this wavefront, when focused, will show the characteristics of an 18-slit grating. All the other wavefronts are formed out of sets of secondary wavelets whose number is less than that of the grating lines, namely, 18. For example, the first order wavefront due to radiation λ_2 consists of 15 wavelets and hence, when focused, will show a characteristic pattern of a 15-slit rather than

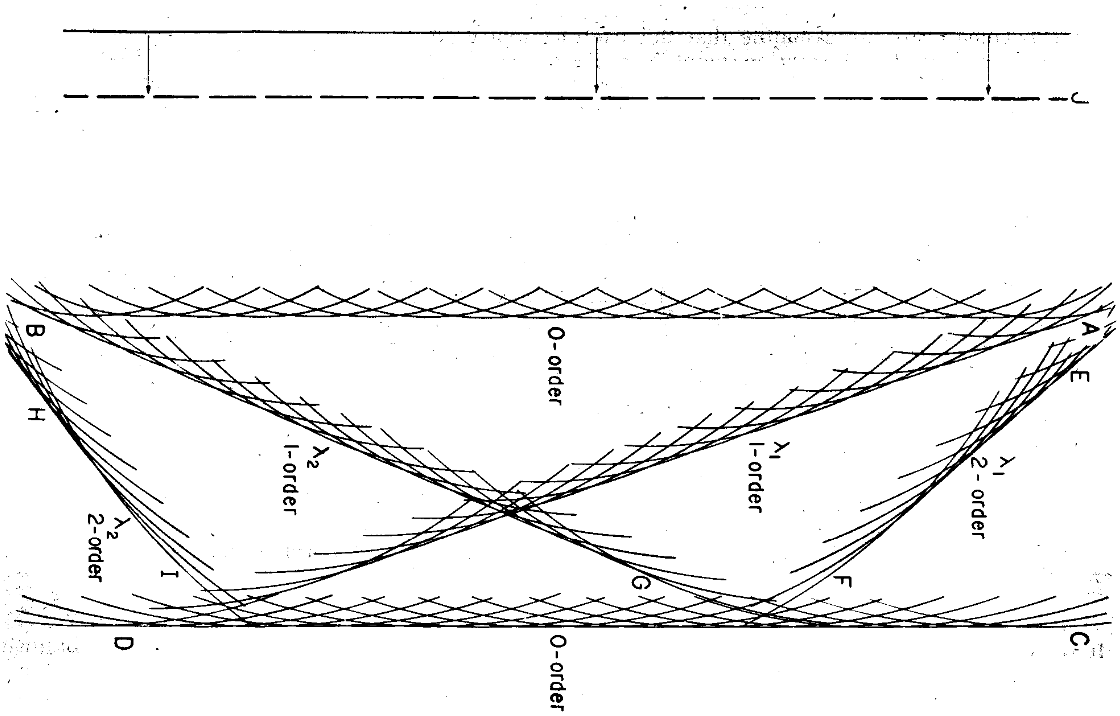

Fig. 3. The formation of the first and second order spectral wavefronts out of Huygens' secondary wavelets produced after a pulse has passed through a grating. In this particular example the pulse of spatial width $AC = BD = 17\lambda_1$ has two radiations, λ_1 and λ_2. The limited width of the pulse causes some of the diffracted wavefronts to be formed only partially.

an 18-slit grating. Thus it is easy to appreciate why the instrumental function, or the grating response curve, depends upon the pulse characteristics and wavelengths of the constituting radiation besides the characteristics of the grating.

3. *Temporal Development Of Echelon Diffraction Patterns*

In this section we develop several particular cases of the transient response of high resolution echelon gratings at the Fourier transform plane due to semi-infinite step, single pulses of different widths, and series of pulses of different widths and separations. In all cases, unless mentioned otherwise, we consider the pulses to be rectangular and to be produced by an ideal ultrafast switch from a stabilized single mode (ν_0) cw laser. As mentioned before, we consider the carrier frequency ν_0 as the only physical frequency of the radiation.

3.1 SEMI-INFINITE STEP

A graphical representation for a semi-infinite step is given in Fig. 4 where the incident semi-infinite step has been replaced by N wavefronts but each delayed from the previous one by the grating order delay-time, $\tau_m = m/\nu_0$. This can be represented by the sum of Eq. 10, but the graphical representation is easier to follow. From the moment of arrival of the first wavefront $(t = 0)$ until $t = \tau_m$, the detector plane receives irradiance due to one wavefront only. Then from $t = \tau_m$ to $t = 2\tau_m$, the irradiance is due to the superposition of two beams generating a two-beam pattern; then from $2\tau_m$ to $3\tau_m$, a three- beam pattern, and so on. The temporal development of the irradiance at an integral order, as would be recorded by an ideal ultrafast detector, is shown in Fig. 5, where the different curves represent the responses at the various bright orders *m-th, 2m-th, 3m-th,* etc. For convenience of plotting, we have assumed that the grating has only 8 slits and the amplitude due to each slit is unity. All the irradiances increase in steps but under a parabolic envelope (see Eq. 12). The horizontal arrows indicate after how much time the particular order reaches the steady-state irra-

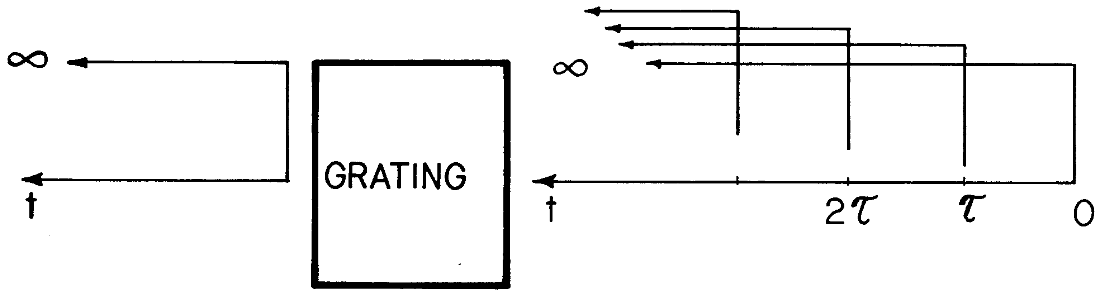

Fig. 4. *Response of a grating to a semi-infinite step pulse.*

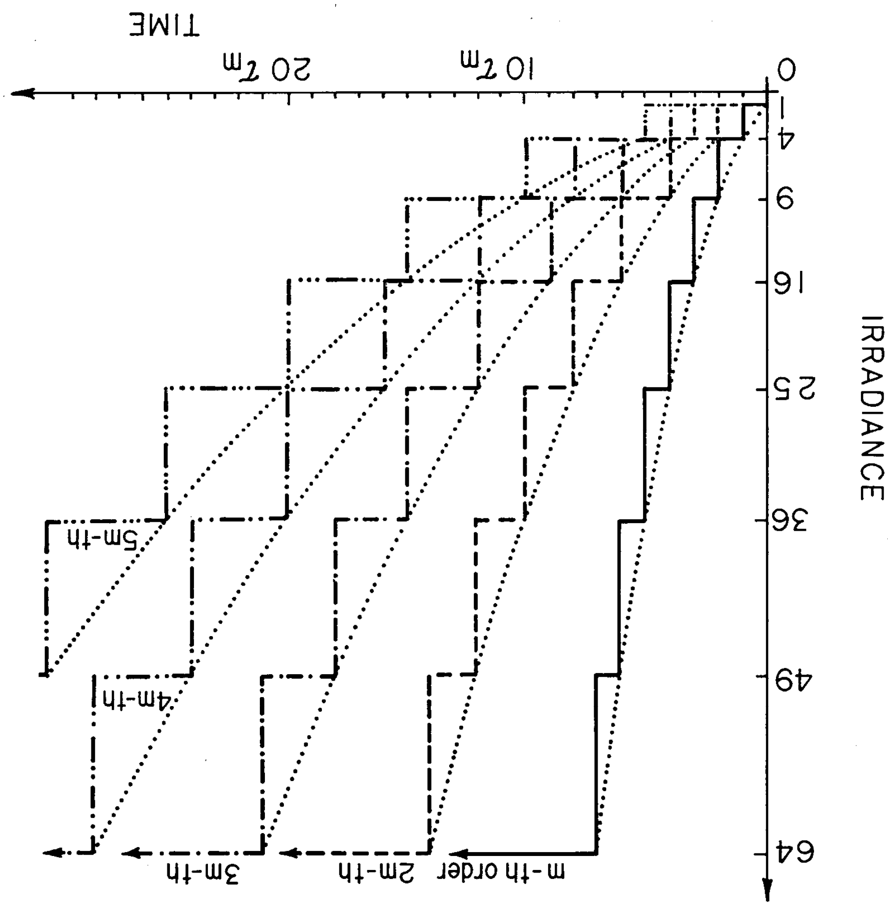

Fig. 5. *Response of a grating to a semi-infinite step pulse; temporal development of irradiances for various integral orders.*

diance. We define this time as response time (or time constant) of the grating. For the *m-th* order we have

$$\tau_r = N\tau_m. \qquad (13)$$

For a plane grating there also exists the zero-order, where all the wavefronts arrive simultaneously. But the zero-order has no dispersive power and is of little interest to a spectroscopist. Let us now look at a half-integral order,

$$\nu_0 \tau = (m + ½), \tag{14}$$

where m is an integer. The irradiance can be found using Eq. 11,

$$|g(t = n\tau_{m+½})|^2 = \frac{\sin^2 \pi\nu_0 n \tau_{m+½}}{\sin^2 \pi\nu_0 \tau_{m+½}} = \sin^2 n\pi/2 \tag{15}$$

where n, the number of superposed beams, varies in steps with time following $t = n\tau_{m+1/2}$. The factor $\sin^2 n\pi/2$ indicates a series of evenly delayed square pulses of light of duration $\tau_{m+1/2}$ seconds separated by dark temporal regimes of equal width as n evolves serially from odd to even integers. This is shown in Fig. 6. Similarly, when steady-state illumination to a grating is suddenly turned off, a series of rectangular pulses should be detected for a period of $N\tau_{m+1/2}$. Thus we see that a grating can be used to produce a series of rectangular pulses of width shorter than the incident pulse. Oscillatory pulses of more complicated nature are produced at positions where the grating order is in between integral and half-integral.

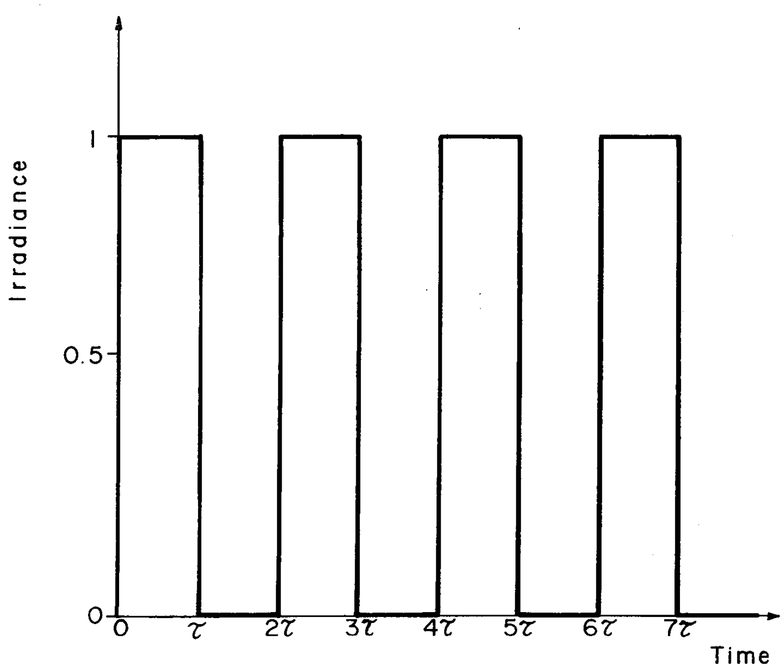

Fig. 6. *Response of a grating to a semi-infinite step pulse at a half-integral order. Theoretically we should be able to detect a series of rectangular pulses all with widths equal to the delay time τ.*

[We now briefly digress to a diffraction pattern due to a beam of quantons and the statistical particle scattering interpretation of quantum mechanics. Physicists generally accept the fact that any interference and diffraction phenomena with light can be completely explained by superposition of waves. Hence the forceful introduction of photons to these phenomena is not only unnecessary but also confusing (Scully and Sargent 1972, Roychoudhuri 1975c, Roychoudhuri et al 1976). However, people introduce photons in diffraction to explain the quantum concept. Probably this is due to the fact that particle diffraction ($\lambda = h/p$) produces patterns which are very similar to those produced by light. This diffraction can be explained by the statistical particle scattering concept using Duane's (1923) momentum quantization rule, as advocated by Landé (1975) and supported by Ballentine (1970). Duane's rule is

$$\Delta p_n = nh/d, \tag{16}$$

where d is the grating period and Δp_n is the quantized momenta exchanged by the particle with the entire grating instantaneously at the moment of interaction. The basic assumption of this paper, that interference is due to real physical superposition of waves at the observation plane, implies a temporal development of the diffraction pattern due to nomena. To test Landé's (1975) argument, let us propose a observation. It also implies that interference is a local phe- different times of arrival of the "wavelets" at the plane of twin experiment with stabilized continuous "waves" of monochromatic light (λ_0) and of monoenergic electrons ($\lambda_0 = h/p$). If a fast detector is set to detect the diffraction pattern from a grating and a shutter is suddenly opened to let the light beam be "scattered" from it, some time varying irradiances should be detected as predicted before. If the same result also happens for the electron beam it would be hard to accept Landé's (1975) argument of instantaneous interaction of an electron with the whole grating, especially because it predicts that no particle should arrive at all if the detector is set at a half-integral order (to be exact, when N is even). The question is, can statistical particle scattering replace the wave-particle duality and explain to phenomenon of diffraction as a pure scattering process?]

3.2 A WIDE RECTANGULAR PULSE

Now consider the response of a grating to a single rectangular pulse. If the order at which the diffracted irradiance is recorded is m, then the corresponding time delay between the consecutive wavefronts from each slit is $\tau_m = m/v_0$ (Eq. 7 or 9). Then, for computational convenience we express the width of the pulse δt in terms of τ_m,

$$\delta t = q\tau_m, \qquad (17)$$

Assume the particular case $q = 6$ and the number of slits in the grating is $N = 8$. The general solution is given by Eqs. 10-12 where $V(t)$ is a rectangular function of width $6\tau_m$ and unit amplitude; it is sketched in Fig. 7. Because we have only 8 slits, there can be only 8 pulses generated by them with delay τ_m and, because the width of the pulse is $6\tau_m$, we cannot have more than 6 pulses superimposed at one time. The peak of the *m-th* order response, solid curve of Fig. 8, shows the evolution of irradiance with time. Figs. 7 and 8 also show the possible solution and response of the grating at the *2m-th* and *3m-th* integral orders.

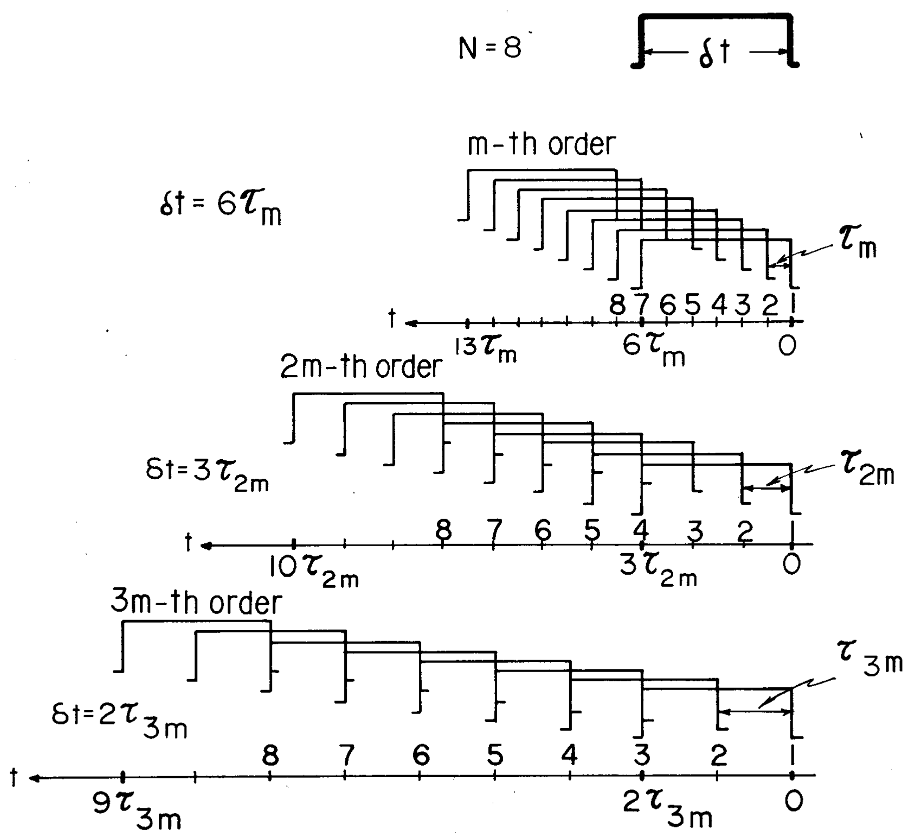

Fig. 7. *Response of a grating to a rectangular pulse. Graphical solution of the problem for various orders.*

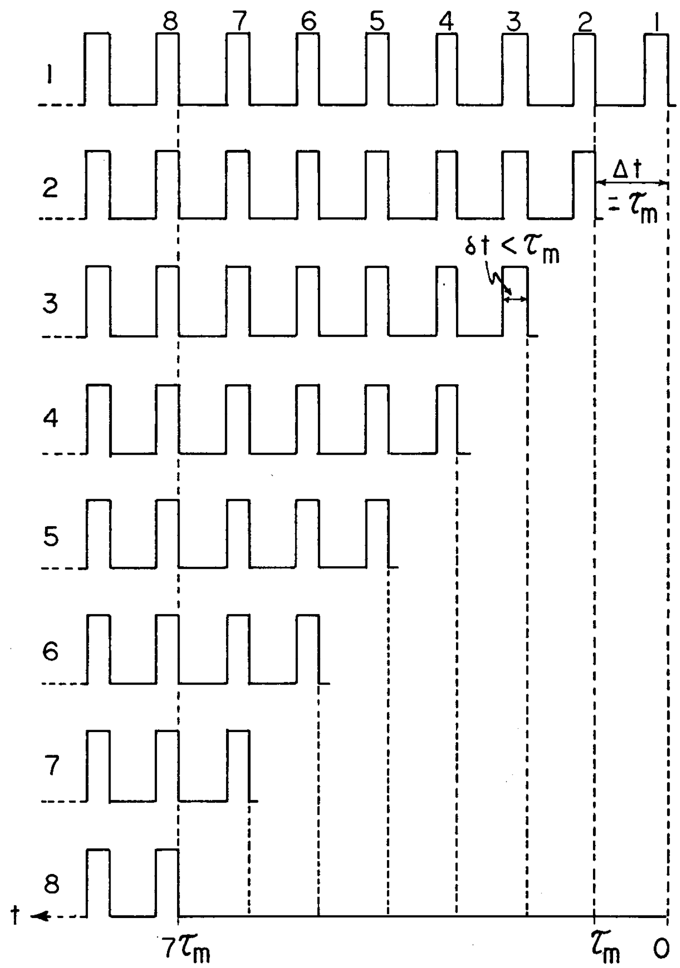

Fig. 12. *Response of a grating to a series of rectangular pulses with pulse separation equal to the delay time for the grating order of interest. Under these conditions an oscillatory "steady" pattern is obtained.*

be eliminated by matching appropriately the pulse separation Δt and the pulse width δt with the grating order. Suppose,

$$\delta t = m/\nu_0, \tag{34}$$

and,

$$\Delta t = pm/\nu_0, \tag{35}$$

where p is any number. When the steady-state has been reached after a time $\tau_r = N\tau_m$, the detector at the m-th order will be receiving constantly the superposition of,

$$N_r = N/p, \tag{36}$$

beams where N_r is defined as the reduced or effective number of grating grooves. Thus the spectroscopic analysis of such pulses from a grating pattern appears to be possible, but the reduced resolving power is,

$$R = mN/p, \tag{37}$$

and the instrumental curve corresponds to a grating pattern of N_r-instead of N-grooves. As a specific example, Fig. 13 shows a graphical solution to the special case $p = 4$ and $N = 8$ which never has more than two-beam interference. Of course, p must be greater than unity to maintain the separation between the individual pulses. Before concluding, we shall briefly comment on the difficulty of spectral analysis of pulses from a mode locked laser. If the number of modes that are locked is n with the frequency of the central mode as ν_0 and the intermode spacing as $\delta\nu$, the width of the main pulse in the train would be (Yariv 1971),

$$\delta t = \frac{2L}{nc} = \frac{1}{n\,\delta\nu}, \tag{38}$$

where L is the length of the laser cavity. The resolving power that is required to resolve these modes is,

$$R_{req} = \frac{\nu_0}{\delta\nu} = \frac{c/\lambda}{c/2L} = \frac{2L}{\lambda}. \tag{39}$$

But the maximum possible resolving power that can be achieved with an isolated single pulse is limited by the very width of the pulse $c\,\delta t$. Then, using (38),

$$R_{max} = \frac{c\delta t}{\lambda_0} = \frac{c/n\delta\nu}{c/\nu_0} = \frac{1}{n}\left(\frac{\nu_0}{\delta\nu}\right) = \frac{R_{req}}{n}. \tag{40}$$

Thus the modes (frequencies) of a single isolated pulse from a mode locked laser can never be resolved in the classical sense. The question now arises as to the possibility of resolving the modes if a whole train of mode locked coherent pulses are used. Now the resolving power would be limited by the path difference (in number of waves) between the first and the last pulse in the train. But, can one observe the n modes spectrally separated with a suitable high resolution spectrometer? It may not be quite possible. This is because when the n modes with a steady phase relation interfere to form mode locked pulses in the laser cavity, the mean frequency (Yariv 1971) ν_0 of the modes becomes the carrier frequency of the pulses and the information corresponding to multiple frequencies has irreversibly changed to amplitude modulation to produce the pulses. We cannot easily unlock them to reproduce n separate frequencies with continuous amplitudes. Still it is worth attempting such an experiment with an arrangement like that of Fig. 13 or its equivalent with a Fabry-Perot (Roychoudhuri 1975a).

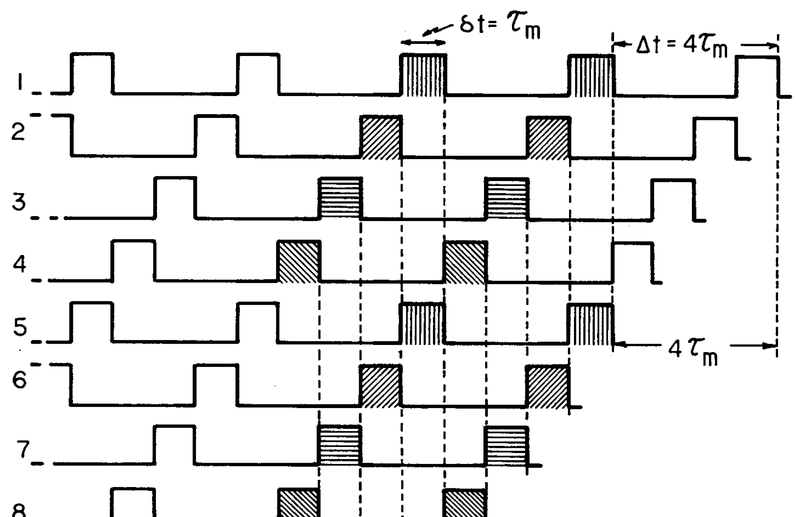

Fig. 13. *Response of a grating to a series of rectangular pulses where the width equals the delay time for the grating order and the separation equals a multiple of the delay time. The response is a steady-state pattern.*

4. *Summary*

To the principle of linear superposition we have added the principle of causality in the sense that the effect cannot be observed until the signal (cause) has arrived at the space and time point under consideration. Neither of these principles are controversial in classical physics. This leads us to believe that the phenomenon of interference is the real physical superposition of the waves carrying different information. Thus two-beam (Milam et al 1974) and multiple-beam (Soures et al 1974, Roychoudhuri 1975a, Thomas and Siebert 1976, Martin and Milam 1976) interferometers give rise to temporally evolving interference patterns because the interfering beams carrying different phase information are temporally delayed from each other.

The main objective of this paper is to extend the above mentioned concept to gratings and generalize it to diffraction phenomenon with the help of Huygens-Fresnel principle that is used in all wave analysis. We have shown that wavefronts from individual grating grooves arrive at the focal plane with a periodic time delay. Hence, gratings produce a time varying pattern like the Fabry-Perot interferometer with pulsed light. This gives rise to time-evolving instrumental response curves (Eqs. 18, 17) that are different than those produced by continuous wave illumination. This property of gratings (Roychoudhuri 1977) and Fabry-Perot's (Thomas and Siebert 1976, Martin and Milam 1976) can be exploited for pulse shaping, pulse generating and pulse width estimation if a fair degree of knowledge of the spectral content of the parent pulse is available. Conversely, on trying to obtain spectral information of an ultrashort pulse by spectrometers, prior knowledge of the pulse shape is essential through other experiments such as two-photon-fluorescence. This helps to compute the pulse-shape and order of interference dependent instrumental response curve that must be deconvolved from the recorded spectrum to obtain the true spectrum of the source pulse. Otherwise, there is a danger of concluding a wider spectral width for the same pulse while analysing it with Fabry-Perot's or gratings of increasing order of interference. This point is apparently ignored by the general literature (Bradley and New 1974) on ultrashort pulse spectroscopy. The problem of spectroscopy is further complicated if the pulse is phase modulated because the interference fringe moves in space with phase modulation.

Further, this analysis (Sec. 3.4) shows that the problems of spectral interpretation can be greatly reduced for a long train of coherent pulses when the pulse separation is an integral multiple of the pulse width that, in turn, is matched with the delay time of a properly chosen grating order (see also Ref. 8). But it is difficult to make a meaningful spectral analysis of mode locked laser pulses. A possible experiment has also been proposed (Sec. 3.1) by which to test whether individual particles exchange momentum with the entire grating instantaneously in particle diffraction experiments (Landé 1975, Roychoudhuri 1975c, Roychoudhuri et al 1976). And we have argued (Sec. 3.2) that the validity of frequency-time indeterminacy principle (Popper 1965, Caulfield et al. 1976) in classical spectroscopy depends upon a general proof that there cannot exist any method to determine the pulse shape exactly without the knowledge of its spectral content.

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5. *Appendix*

In section 2 we have developed the time varying diffraction pattern for gratings with extremely narrow slits and for wide step echelon gratings where only the step delay in the arrival of the signals from different slits (steps) has been taken into account. We did not take into account the continuous variation in the arrival time of the signals from the continuously distributed points within the same slit. But such a time variation is not of great consequence for visible light pulses of width up to about tenth of a picosecond (10^{-13} sec.). This can be appreciated from the fact that a first order diffraction implies a path delay of λ and hence a time delay of

$$\lambda/c = \nu^{-1} \sim 10^{-15} \text{ sec.} \qquad (38)$$

But, for the sake of completeness and in anticipation that a few femtosecond pulses may be produced in the future, we shall develop here the space-time diffraction pattern due to an aperture.

Established diffraction theories (Goodman 1968, Born and Wolf 1975) give us the Fraunhofer pattern $u(x, y, t)$ due to an aperture $U(\xi, \eta)$ when illuminated by a continuous plane wave of single frequency $\exp\ (2\pi i\nu_0 t)$,

$$u(x, y, t) = Re\ \frac{\nu_0 A}{2ic}\ e^{ikZ_0}\ e^{2\pi i\nu_0 t} \int d\xi \int d\eta\ U(\xi, \eta)\ e^{-2\pi i\nu_0(\xi x+\eta y)/fc}, \qquad (39)$$

where Re implies the real part of the entire expression that gives the instantaneous amplitude, Z_0 is the axial optical delay between OO' (Fig. 14) and $(\xi x + \eta y)/f$ is the optical delay or the perpen-

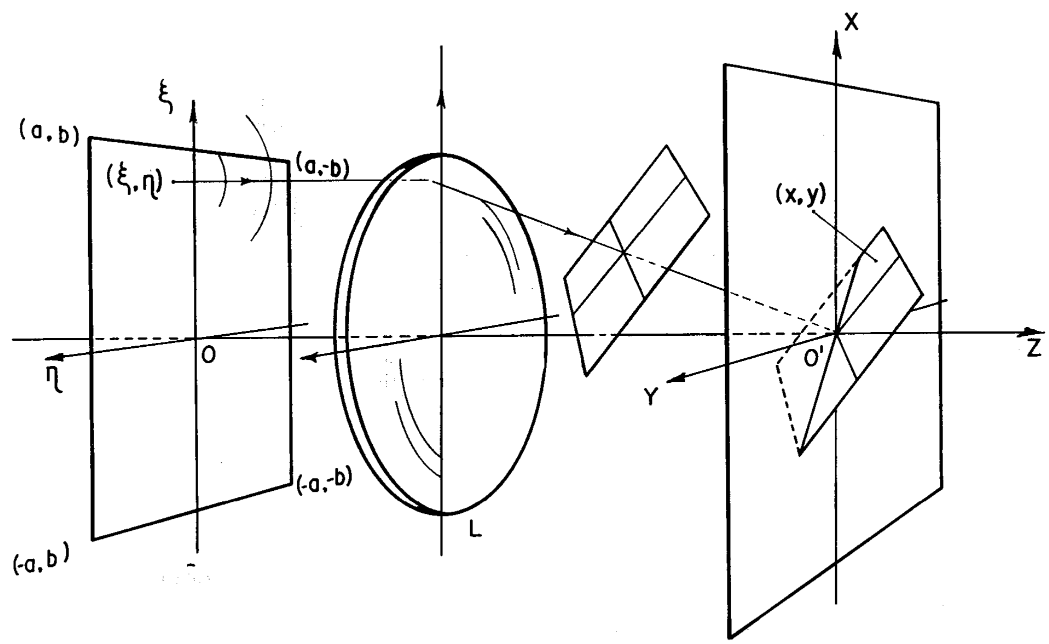

Fig. 14. *Derivation of space-time diffraction pattern by an aperture. L is the transforming lens; the front focal plane* (ξ-η) *is the aperture plane and the rear focal plane (x-y) is the observation plane.*

dicular distance from the point (x, y) of the inclined plane wavelet (crossing O') due to the spherical wavelet originated at the point (ξ, η). The last physical interpretation is more easily appreciated for the one dimensional case. In order words, besides the overall delay from O to O' of $\tau_0 = Z_0/c$, the signals originating from different points (ξ, η) arrive at the observation point (x, y) with variable delays t, given by

$$t(\xi, \eta, x, y) = (\xi x + \eta y)/fc. \tag{40}$$

For a continuous radiation illuminating the aperture, such an optical time delay implies only a phase delay factor as in Eq. 39 without any time dependence and hence one can integrate over the entire aperture. But if we modulate the illuminating amplitude exp $(2\pi i\nu_0 t)$ by a time function $V(t)$, the phase factor in Eq. 39 or 40 is time-dependent and hence the integration limits cannot extend over the entire aperture at a time. This is because, while signals from some parts of the aperture have already arrived at the (x, y) point, some are still on their way from other parts of the same aperture. Further, these amplitudes are multiplied not only by the aperture function $U(\xi, \eta)$, but also by the time function $V(t)$. Then the space-time diffraction pattern $u(x, y, t)$ should be written as,

$$u(x, y, t) = Re\, D\, e^{2\pi i\nu_0 t} \int_{\xi_0}^{\xi(t)} d\xi \int_{\eta_0}^{\eta(t)} d\eta\, U(\xi, \eta)\, V(t) e^{-2\pi i\nu_0(\xi x+\eta y)/fc} \tag{41}$$

where D represents all the constants before the integral of Eq. 39, (ξ_0, η_0) is the point from which the light signal arrives earliest ($t = 0$ for the observer) at the observation point (x, y) and $(\xi(t), \eta(t))$ is the point from which the signals have just arrived at a time t given by Eq. 40. It is to be noted that for a given time and observation point, Eq. 40 defines a straight line and hence all the signals which originate on this line arrive simultaneously at (x,y). Thus the whole aperture can be imagined to be built by laying very many parallel lines represented by the following intercept form,

$$\frac{\xi}{(fct/x)} + \frac{\eta}{(fct/y)} = 1. \tag{42}$$

The amplitude contribution due to each of these lines will depend upon the length of the lines intercepted by the aperture $U(\xi, \eta)$.

We shall simplify our example by considering a rectangular amplitude slit with $U(\xi, \eta) = 1$ within the rectangle 2a by 2b. A second simplification is achieved by considering the point of observation on the X-axis (i. e., $y = 0$). Then Eq. 40 indicates that the time delay is independent of η-coordinate. This means that for a given ξ all the "source points" lying on a line parallel to η-axis will arrive at x at the same time. A further simplification can be achieved by assuming the incident pulse $V(t)$ to be a rectangular one of width δt and of unit amplitude. Then the space-time diffraction pattern at x for a time interval $t \leqslant \delta t$ is,

$$u(x, 0, t) = Re\ D\ e^{2\pi i \nu_0 t} \int_a^{\xi(t)} d\xi\ e^{-2\pi i \nu_0 \xi x / fc} \int_{-b}^{b} d\eta$$

$$= Re\ D2b\ e^{2\pi i \nu_0 t} [\xi(t) - a] \operatorname{sinc} \frac{kx}{2f} [\xi(t) - a]\ e^{-\frac{ikx}{2f}[\xi(t) + a]} \qquad (43)$$

The physical effect of this time dependent diffraction pattern is as if the aperture is slowly opening up from one end to the other. If the pulse width δt is much larger than the time delay in arrival of signals from +a to −a, i. e., if,

$$\delta t \gg \frac{2ax}{fc} \equiv \tau_c, \qquad (44)$$

the pattern will become steady after a time τ_c (the time constant of the aperture) because $\xi(t)$ could be replaced by −a in Eq. 43. The pattern will become time varying again for an interval τ_c before completely disappearing. Finally, if the incident pulse has more than one incoherent carrier frequency, the Eq. 41 should be integrated over these frequencies after taking its square modulus.

Our formulation of the space-time diffraction phenomenon and the corresponding prediction of particular results are quite different from the standard formulation that can be found in texts (Goodman 1968, Born and Wolf 1975) and recent publications (Froehly et al 1973, Vienot et al 1977). The standard formulation follows the following steps. The time amplitude modulation function $V(t)$ is replaced by the time independent Fourier frequency integral as a summation of many continuous monochromatic radiations. The limits of the space aperture integral cover the entire aperture rather than time varying limits as we have used. Then the Fourier frequency integral is associated with the constant factor ν_0 (Eq. 39) obtained from the diffraction formulation based on Huygens-Fresnel principle; thus equating the carrier physical frequency ν_0 of the radiation with the purely mathematical Fourier frequency ν. But this helps one to rewrite the Fourier integral as the time derivative of the incident pulse $V(t)$. The final result of this unphysical mathematical trick is that the space-time diffraction integral Eq. 41 becomes a space integral over the entire aperture that is multiplied by the time derivative of the amplitude modulation factor $V(t)$ (Goodman 1968). That this result is non-physical can be easily verified by taking a few simple counter examples like $V(t) =$ constant, Ct, a rectangular function, etc. The time derivates of these pulses are respectively zero, constant and a pair of delta functions with opposite signs. Thus, when the incident pulse is a square one, we are supposed to have a pair of delta function type response irrespective of the length of the rectangular pulse. Whereas we know that the result should converge to the stationary illumination when the pulse is very long as predicted by our causal formulation (Eq. 43). A further mathematical trick (Froehly et al 1973, Fouéré 1976, 1977, Vienot et al 1977) gives a modified result: "the time response attached to a given geometrical pupil is the convolution of the temporal input function with the first time derivative of the projection of the pupil along the considered direction of diffraction" (Vienot et al 1977). This can also be shown to be wrong by taking a simple counter example like a rectangular aperture and considering the diffraction in the forward direction.

We would also like to mention the recent article (Eberly and Wódkiewicz 1977) on the time dependent physical spectrum of light where the apparent spirit of the authors is to avoid defining spectrum based on any Fourier transformation. To this spirit we agree very much (Roychoudhuri 1976a). But they define the spectrum of a time-pulse based on the Fabry-Perot response curve. This is not proper from the stand point of Physics. Then other spectrometers like prisms and gratings with different instrumental response functions will give different spectrum for the same pulse. Further, the authors take the Fourier transform of the ideal Airy function of a Fabry-Perot as the time impulse response. Thus they are violating causality through the use of non-causal Fourier integral and thereby going against the spirit of their own paper. The time impulse response should be directly obtained as the sum of the train of the replicated pulses as in Roychoudhuri (1975a) Ref. 8 or Eq. 10 of this paper. And one will find different intensity response curves for the same pulse with the same carrier frequency ν_0 for different orders of interference as shown in Figs. 10 and 11. Do they correspond to different physical spectrum?

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